Finite Difference Method for the Estimation of a Heat Source Dependent

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Abstract

The inverse parabolic problem is of significant importance in mathematical sciences, applied sciences and engineering [1-3]. In the process of transportation, diffusion and conduction of natural materials, the parabolic partial differential equation is induced (see [4]). In this paper, we present the numerical solutions of the parabolic inverse problem with the Dirichlet condition. Problem requires finding the temperature $u(x,t)$ and the unknown right hand side term $p(t)$ satisfying the heat equation

$$u_t - u_{xx} + u = p\left(t\right)q\left(x\right) + f\left(t,x\right),\ \text{ in } (x,t) \in (0,L) \times (0,T] \tag{1}$$

subject to the initial condition

$$u(x,0) = u_0(x), 0 \leq x \leq L, \tag{2}$$

the Dirichlet boundary condition

$$u(0,t) = u\left(L,t\right) = 0, 0 < t \leq T, \tag{3}$$

and the overdetermined conditions in an interior point

$$u(x^*,t) = 0, 0 < t \leq T,\ 0 < x^* < L. \tag{4}$$

The coercive stability estimates for the solution of first and second orders of accuracy difference scheme are established in the difference analogue of a space of smooth functions. The theoretical statements for the solution of this difference schemes for parabolic inverse problem is supported by the results of numerical experiments.

References

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