Some investigation of the numerical solution of ordinary differential equations

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Abstract

With the numerical solution of ordinary differential equations (ODE), scientists engaged in the Middle Ages, beginning with the work of Clairaut. The domain of the numerical methods involved in many famous mathematicians - Euler, Runge, Kutta, Adams, Laplace, and others. They have constructed methods with different properties.

In this paper we consider the construction of numerical methods with high accuracy and to this end is proposed to use multistep multiderivative methods. As well as specific methods are constructed with degree $p \leq 8$.

It is known that many problems of natural science are reduced to ordinary differential equations. There are numerous works devoted to the numerical solution of ordinary differential equations, among which are the popular Runge-Kutta and Adams. These methods are development and generalization of Euler's method. Researchers assessed these methods in different ways and in the middle of the twentieth century, constructed methods that have better properties of Runge-Kutta and Adams, who called hybrid. Here, the numerical solution of the following initial value problems:

$$y' = f(x, y), \quad y(x_0) = y_0, \quad x_0 \leq x \leq X. \tag{1}$$

is suggested to apply the forward-jumping method of running too far ahead and the hybrid method. We assume that problem (1) has a unique solution $y(x)$ defined on an interval $[x_0, X]$. To determine the approximate values of the solutions of (1), the segment $[x_0, X]$ with a constant step $h > 0$ is divided into N equal parts, and the mesh points denoted by $x_i = x + ih (i = 0, 1, 2, ..., N)$. We denote the approximate value y_i and y'_i the solution and its derivatives at a point $x_i = x_0 + ih (i = 0, 1, 2, ..., N)$, but the exact values of the solution and its first derivative at the point $x_i = x_0 + ih (i = 0, 1, 2, ..., N)$ a $y(x_i)$ and $y'(x_i)$.

The forward-jumping method in the simplest case can be written as follows:

$$\sum_{i=0}^{k-m} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i^j f_{n+i}, \quad (m \geq 0), \tag{2}$$

but a hybrid method in one variant, can be written as:

$$\sum_{i=0}^{k} \alpha_i y_{n+i} = h \sum_{i=0}^{k} \beta_i f_{n+i} + h \sum_{i=0}^{k} \gamma_i f_{n+i+\nu_i} \quad (|\nu_i| < 1; i = 0, 1, 2, ...). \tag{3}$$

Here construct concrete hybrid methods, as well as methods of type forward-jumping method with the degree $p \leq 6$, an algorithm for using the constructed methods.