Positivity of Elliptic Difference Operators and its Applications

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As is well-known that the investigation of well-posedness of various types of parabolic and elliptic differential and difference equations is based on the positivity of elliptic differential and difference operators in various Banach spaces and on the structure of the fractional spaces generated by these positive operators. An excellent survey of works in the theory of fractional spaces generated by positive multidimensional difference operators in the space and its applications to partial differential equations was given in [1]-[2]. In a number of works (see, e.g., [3]-[11], and the references therein) difference schemes were treated as operator equations in a Banach space and the investigation was based on the positivity property of the operator coefficient.

In the present paper, we consider the difference operator

$$(-1)^n \partial_{h_p}^{2n} + A_h,$$

where A_h is the self-adjoint positive definite operator in L_{2h}. Applying the method of paper [3] the positivity of this difference operator in the Holder spaces is established. In applications, the well-posedness of the Cauchy problem for parabolic differential and difference equations is investigated.

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