Oscillation Theorems for Second-Order Damped Dynamic Equation on Time Scales

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Abstract

Much recent attention has been given to dynamic equations on time scales, or measure chains, and we refer the reader to the landmark paper of S. Hilger [1] for a comprehensive treatment of the subject. A book on the subject of time scales by Bohner and Peterson [2] also summarizes and organizes much of the time scale calculus.

In this paper we shall study the oscillations of the following nonlinear second-order dynamic equations with damping

$$(r(t)\Psi(x^{\Delta}(t))^{\Delta} + p(t)\Psi(x^{\Delta}(t)) + q(t)f(x^{\sigma}(t)) = 0,\ t \in \mathbb{T}, \tag{1}$$

where $\Psi(t)$, $f(t)$, $p(t), q(t)$ and $r(t)$ are rd-continuous functions. By using a generalized Riccati transformation and integral averaging technique, we establish some new sufficient conditions which ensure that every solution of this equation oscillates. Throughout this paper, we will assume the following hypotheses:

(H_1) $p(t),\ q(t) \in C_{rd}(\mathbb{R}, \mathbb{R}^{+})$,

(H_2) $\Psi : \mathbb{T} \to \mathbb{R}$ is such that $\Psi^2(u) \leq \kappa u \Psi(u)$ for $\kappa > 0,\ u \neq 0$,

(H_3) $f : \mathbb{R} \to \mathbb{R}$ is such that $\frac{f(u)}{u} \geq \lambda > 0$, and $uf(u) > 0,\ u \neq 0$,

(H_4) $r(t) \in C^1_{rd}([t_0, \infty), \mathbb{R}^{+}),\ \int_{t_0}^{\infty} (\frac{1}{r(t)} e_{\frac{-p(t)}{r(t)}}(t, t_0))\Delta t = \infty$.

References

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