FIRST INTERNATIONAL CONFERENCE ON ANALYSIS AND APPLIED MATHEMATICS

ICAAM 2012

Gumushane, Turkey   18 – 21 October 2012

ABSTRACT BOOK

ICAAM2012

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Shadowing in the parabolic equations

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This talk is devoted to the numerical analysis of abstract semilinear parabolic problem $u'(t) = Au(t) + f(u(t)), u(0) = u^0$, in some general Banach space E. We are developing a general approach to establish the discrete dichotomy in a very general setting and prove shadowing Theorems that compare solutions of the continuous problem with those of discrete approximation in time. It is well-known fact that the phase space in the neighborhood of the hyperbolic equilibrium can be split in a such way that the original initial value problem is reduced to initial value problems with exponential decaying solutions in opposite time direction. We use the theory of compact approximation principle and collectively condensing approximation to show that such a decomposition of the flow persists under rather general approximation schemes. The main assumption of our results are naturally satisfied, in particular, for operators with compact resolvents and condensing semigroups and can be verified for finite element as well as finite difference methods.

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Fractals arising from Newton's method

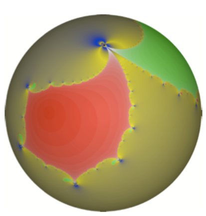
Abstract. The aim of this talk is to introduce the concept of fractals arising from Newton's method. We consider the dynamics as a special class of rational functions that are obtained from Newton's method when applied to a polynomial equation. Finding solutions of these equations leads to some beautiful images in complex functions. These images represent the basins of attraction of roots of complex functions. If z_0 is an attracting periodic point of some rational function of degree larger that one, its basin of attraction is as follows:

$$\mathcal{B}(z_0) := \{z \in \mathbb{C} \,|{N_f}^n(z_0) \text{ converges to } z_0,\ n \to \infty\}.$$

The basin of attraction $\mathcal{B}(z_0)$ is a union of components of the Fatou set, and the boundary of $\mathcal{B}(z_0)$ coincides with the Julia sets of a rational function N_f. In this presentation, we seek will the answer of the following question:

"What is the dynamics near the chosen parabolic fixed points?"

For example,



$f(z) = (z^2+4)e^z$ the Newton function of f is $N_f(z) = \frac{z^3+z^2+4z-z}{z^2+2z+4}$ and the fractal image of that function on Riemann sphere is presented.

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On the Classifications of C^*-Algebras Using Unitary Groups

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Abstract

In 1955, Dye proved that the discrete unitary group in a factor determines the algebraic type of the factor. Using Dye's approach, we prove similar results to a larger class of amenable unital C^*-algebras including simple unital AH-algebras (of SDG) with real rank zero. If φ is an isomorphism between the unitary groups of two unital C^*-algebras, it induces a bijective map θ_φ between the sets of projections of the algebras. For some UHF-algebras, we construct an automorphism φ of their unitary group, such that θ_φ does not preserve the orthogonality of projections. For a large class of unital C^*-algebras, we show that θ_φ is always an orthoisomorphism. This class includes in particular the Cuntz algebras $\mathcal{O}_n$, $2 \leq n \leq \infty$, and the simple unital AF-algebras having 2-divisible K_0-group. If φ is a continuous automorphism of the unitary group of a UHF-algebra A, we show that φ is implemented by a linear or a conjugate linear $*$-automorphism of A.

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Solution of One Problem for the Equation of Parabolic type with Involution Perturbation

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Abstract. In the domain $D = \{(x,t) : -1 < x < 1, 0 < t < T\}$ the following problem is considered: Obtain the solution $u \in C^{2,1}(D) \cap C(D)$ of the equation

$$u_t(x,t) = u_{xx}(-x,t) - \alpha u_{xx}(x,t), \tag{1}$$

which satisfies the conditions

$$u(x,0) = \varphi(x), -1 \leq x \leq 1; u(-1,t) = 0, u(1,t) = 0, 0 \leq t \leq T, \varphi(-1) = 0, \varphi(1) = 0. \tag{2}$$

Application of the Fourier method gives the spectral problem of the form

$$-X''(-x) + \alpha X''(x) = \lambda X(x), -1 < x < 1, X(-1) = 0, X(1) = 0. \tag{3}$$

The system of eigenfunctions of (3) is generated Riesz basis in $L_2(-1,1)$.

Theorem 1. If α is real number α and $|\alpha| > 1$, then the problem (1)-(2) has a unique solution.

Theorem 2. If α is a complex number and $|Re\alpha| \geq 1$, then the problem (1)-(2) has a unique solution.

Moreover, we have the following formula

$$u(x,t) = \sum_{k=0}^{\infty} c_k e^{(1-\alpha)(\frac{\pi}{2}+k\pi)^2 t} \cos(\frac{\pi}{2} + k\pi)x + \sum_{k=1}^{\infty} d_k e^{-(1+\alpha)(k\pi)^2 t} \sin k\pi x.$$

Many papers are devoted to investigation of partial differential equations and spectral problems of differential operators with involution see, for example, [1-4].

Keywords: Spectral of Problems, Differential Operators with involutions, Exact Solutions, Fourier Series
PACS: 87.10.Ed

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A Note on the Numerical Solution of Fractional Schrödinger Differential Equations

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Abstract

Many different equations are called by fractional Schrödinger differential equation (FSDE) until today. In recent years, the FSDE which is derived from classical Schrödinger differential equation has received more attention. This problem is solved by some numerical methods (see [2]-[5]). However, finite difference method which is a useful tool for investigation of fractional differential equations has not been applied to a FSDE yet. The present paper fills a gap by applying finite difference method to the following multi-dimensional linear FSDE

$$\begin{cases} i\frac{\partial^\alpha u(t,x)}{\partial t^\alpha} - \sum\limits_{r=1}^{m} (a_r(x)u_{x_r})_{x_r} + \delta u(t,x) = f(t,x), \\ 0 < t < 1, x = (x_1, \cdots, x_m) \in \Omega, \\ u(0,x) = 0,\ x \in \overline{\Omega}, \\ u(t,x) = 0,\ x \in S \end{cases} \tag{1}$$

where $0 < \alpha < 1$. Here $a_r(x),\ x \in \Omega$ and $f(t,x)$ $(t \in [0,1],\ x \in \Omega)$ are given smooth functions and $a_r(x) \geq a \geq 0$. First and second orders of accuracy difference schemes are constructed for problem (1). Numerical experiment on a one-dimensional FSDE shows the effectiveness of the difference schemes.

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On Bitsadze-Samarskii type nonlocal boundary value problems for semilinear elliptic equations

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Abstract

In the literature, the problem of Bitsadze-Samarskii type is often referred to as the boundary value problem with Bitsadze-Samarskii condition (see [2], [4] and [7]). Previously, the Bitsadze-Samarskii type nonlocal boundary value problems for linear elliptic equations were studied ([5]). In this paper, the Bitsadze-Samarskii type nonlocal boundary value problems for semilinear elliptic equations

$$\begin{cases} -\frac{d^2u(t)}{dt^2} + Au(t) = f(t,u(t)), 0 < t < 1, \\ u(0) = \varphi,\ u(1) = \sum\limits_{j=1}^{J} \alpha_j u(\lambda_j) + \psi, \\ 0 < \lambda_1 < \cdots < \lambda_J < 1, \sum\limits_{j=1}^{J} |\alpha_j| \le 1 \end{cases}$$

in a Hilbert space H with the self-adjoint positive definite operator A is considered. The first and second orders of accuracy difference schemes approximately solving these problems are studied. A procedure of modified Gauss elimination method is used for solving these difference schemes for the two-dimensional elliptic differential equation. The method is illustrated by numerical examples. The converge estimates for the solution of these difference schemes are obtained.

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A Third-Order of Accuracy Difference Scheme for the Bitsadze-Samarskii Type Nonlocal Boundary Value Problem

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Abstract

The role played by coercive inequalities in the study of local boundary-value problems for elliptic and parabolic differential equations is well-known ([1], [2]). Theory, applications and methods of solutions of Bitsadze-Samarskii nonlocal boundary value problems for elliptic differential equations have been studied extensively by many researchers ([3]-[5]). The Bitsadze-Samarskii type nonlocal boundary value problem

$$\begin{cases} -\frac{d^2u(t)}{dt^2} + Au(t) = f(t),\ 0 < t < 1, \\ u_t(0) = \varphi, u_t(1) = \beta u_t(\lambda) + \psi, \\ 0 \le \lambda < 1,\ |\beta| \le 1 \end{cases}$$

for the differential equation in a Hilbert space H with the self-adjoint positive definite operator A is considered. The third order of accuracy difference scheme for the approximate solution of this problem is presented. The well-posedness of this difference scheme in difference analogue of Hölder spaces is established. In applications, the stability, the almost coercivity and the coercivity estimates for solution of difference scheme for elliptic equations are obtained.

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NBVP for Hyperbolic Equations Involving Multi-point and Integral Conditions

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Abstract

Nonlocal boundary value problems involving multi-point and integral conditions for a hyperbolic equation in a Hilbert space are investigated. The stability estimates for the solution of these

multi-point NBVP are established. In applications, the stability estimates for the solution of these problems are obtained.

The authors of [3] developed a numerical procedure for the NBVP with a integral conditions for hyperbolic equations. In the paper [4], instead of nonlocal integral conditions multi-point nonlocal conditions used. In the present work, we consider the NBVP with multi-point and integral conditions

$$\begin{cases} \frac{d^2u(t)}{dt^2} + Au(t) = f(t) \quad (0 \leq t \leq 1), \\ u(0) = \int\limits_0^1 \alpha(\rho) u(\rho) d\rho + \sum\limits_{i=1}^{n} a_i u(\lambda_i) + \varphi, \\ u_t(0) = \int\limits_0^1 \beta(\rho) u_t(\rho) d\rho + \sum\limits_{i=1}^{n} b_i u_t(\lambda_i) + \psi \end{cases} \tag{1}$$

for the differential equation in a Hilbert space H with a self-adjoint positive definite operator A. We are interested in studying the stability of solutions of problem (1) under the assumption

$$\left| 1 + \int\limits_0^1 \alpha(s)\beta(s)\,ds + \sum_{k=1}^{n} a_k b_k + \sum_{k=1}^{n} a_k \int\limits_0^1 \beta(s)\,ds + \sum_{k=1}^{n} b_k \int\limits_0^1 \alpha(s)ds \right| \tag{2}$$

$$> \int\limits_0^1 (|\alpha(s)| + |\beta(s)|)\,ds + \sum_{k=1}^{n} |a_k + b_k|\,.$$

A function $u(t)$ is a *solution* of problem (1) if the following conditions are satisfied:

i) $u(t)$ is twice continuously differentiable on the interval $(0,1)$ and continuously differentiable on the segment $[0,1]$. The derivatives at the endpoints of the segment are understood as the appropriate unilateral derivatives.

ii) The element $u(t)$ belongs to $D(A)$ for all $t \in [0,1]$, and function $Au(t)$ is continuous on the segment $[0,1]$.

iii) $u(t)$ satisfies the equation and nonlocal boundary conditions (1).

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Boundary Value Problem for a Third Order Partial Differential Equation

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Abstract. Boundary value problems for third order partial differential equations in a Hilbert space are investigated. The stability estimates for the solution of the boundary value problem is established. To validate the main result, some stability estimates for solutions of the boundary value problems for third order equations are given. Here, the boundary value problem

$$\begin{cases} \frac{d^3u(t)}{dt^3} - Au(t) = f(t),\ 0 < t < 1, \\ u(0) = \varphi,\, u_t(0) = \psi,\, u_{tt}(1) = \xi, \end{cases} \tag{1}$$

for a third order partial differential equation in a Hilbert space H with a self-adjoint positive definite operator A is considered. We are interested in studying the stability of solutions of problem (1).

A function $u(t)$ is a *solution* of problem (1) if the following conditions are satisfied:

i) $u(t)$ is three times continuously differentiable on the interval $(0,1)$ and continuously differentiable on the segment $[0,1]$. The derivatives at the endpoints of the segment are understood as the appropriate unilateral derivatives.

ii) The element $u(t)$ belongs to $D(A)$ for all $t \in [0,1]$, and function $Au(t)$ is continuous on the segment $[0,1]$.

iii) $u(t)$ satisfies the equation and boundary conditions (1).

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Fractional Parabolic Differential and Difference Equations with the Dirichlet-Neumann Condition

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Abstract

The multidimensional fractional parabolic equation with the Dirichlet-Neumann condition is studied. Stability estimates for the solution of the initial-boundary value problem for this fractional parabolic equation are established. The stable difference schemes for this problem are presented. Stability estimates for the solution of the first order of accuracy difference scheme are obtained. A procedure of modified Gauss elimination method is applied for the solution of first and second order of accuracy difference schemes of one-dimensional fractional parabolic differential equations.

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High Order of Accuracy Stable Difference Schemes for Numerical Solutions of NBVP for Hyperbolic Equations

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Abstract

The abstract nonlocal boundary value problem for the hyperbolic equation

$$\begin{cases} u^{''}(t) + Au(t) = f(t), \ 0 < t < T, \\ u(0) = \alpha u(1) + \varphi, \ u'(0) = \beta u'(1) + \psi \end{cases}$$

in a Hilbert space H with the self -adjoint positive definite operator A is considered. The third and fourth order of accuracy difference schemes for the approximate solutions of this problem are presented. The stability estimates for the solutions of these difference schemes are obtained and numerical results are presented in order to verify theoretical statements.

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Positivity of Two-dimensional Elliptic Differential Operators in Hölder Spaces

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Abstract

This paper considers the following operator

$$Au(t,x) = -a_{11}(t,x)u_{tt}(t,x) - a_{22}(t,x)u_{xx}(t,x) + \sigma u(t,x),$$

defined over the region $\mathbb{R}^{+} \times \mathbb{R}$ with the boundary condition $u(0,x) = 0$, $x \in \mathbb{R}$. Here, the coefficients $a_{ii}(t,x)$, $i = 1,2$ are continuously differentiable and satisfy the uniform ellipticity

$$a_{11}^{2}(t,x) + a_{22}^{2}(t,x) \geq \delta > 0,$$

and $\sigma > 0$. It investigates the structure of the fractional spaces generated by this operator. Moreover, the positivity of the operator in Hölder spaces is proved.

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On the Numerical Solution of Ultra Parabolic Equations with Neumann Condition

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Abstract

In this paper, our interest is studying the stability of first order difference scheme for the approximate solution of the initial boundary value problem for ultra parabolic equations

$$\begin{cases} \frac{\partial u(t,s)}{\partial t} + \frac{\partial u(t,s)}{\partial s} + Au(t,s) = f(t,s), \quad 0 < t, s < T, \\ u(0,s) = \psi(s), \quad 0 \le s \le T, \\ u(t,0) = \varphi(t), \quad 0 \le t \le T \end{cases} \tag{1}$$

in an arbitrary Banach space E with a strongly positive operator A.We refer to [1, 2] and the references therein for a series of papers by the authors, dealing with ultra parabolic equations , arising in diffusion theory, probability and finance. Some new results about numerical methods for ultra-parabolic equations are also announced, see [3-5]. For approximately solving problem (1), the first-order of accuracy difference scheme

$$\begin{cases} \frac{u_{k,m}-u_{k-1,m}}{\tau} + \frac{u_{k-1,m}-u_{k-1,m-1}}{\tau} + Au_{k,m} = f_{k,m} \ , \\ f_{k,m} = f(t_k, s_m),\ t_k = k\tau,\ \ s_m = m\tau,\ 1 \le k, m \le N,\ N\tau = 1, \\ u_{0,m} = \psi_m, \psi_m = \psi(s_m),\ 0 \le m \le N, \\ u_{k,0} = \varphi_k, \varphi_k = \varphi(t_k),\ 0 \le k \le N \end{cases} \tag{2}$$

is presented. The stability estimates and almost coercive stability estimates for the solution of difference schemes (2) is established. In applications, the stability in maximum norm of difference shemes for multidimensional ultra parabolic equations with Neumann condition is established. Applying the difference schemes, the numerical methods are proposed for solving one dimensional ultra parabolic equations.

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Existence and Uniqueness of Solutions for Nonlinear Impulsive Differential Equations with Two-point and Integral Boundary Conditions

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Abstract

The theory of impulsive differential equations is an important branch of differential equations, which has an extensive physical background. Impulsive differential equations arise frequently in the modeling many physical systems whose states are subjects to sudden change at certain moments. There has a significant development in impulsive theory especially in the area of impulsive differential equations with fixed moments; see for instance the monographs [1-4] the references therein.

In this paper, the sufficient conditions are established for the existence of solutions for a class of two-point and integral boundary value problems for impulsive differential equations.

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Optimal Control Problem for Impulsive Systems with Integral Boundary Conditions

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Abstract

Impulsive differential equations have become important in recent years as mathematical models of phenomena in both physical and social sciences. There is a significant development in impulsive theory especially in the area of impulsive differential equations with fixed moments; see for instance the monographs [1-4] and the references therein.

Many of the physical systems can be described better by integral boundary conditions. Integral boundary conditions are encountered in various applications such as population dynamics, blood flow models, chemical engineering and cellular systems. Moreover, boundary value problems with integral conditions constitute a very interesting and important class of problems. They include two, three, multi and nonlocal boundary value problems as special cases, (see [5-7]). For boundary value problems with nonlocal boundary conditions and comments on their importance, we refer the reader to the papers [8-10] and the references therein.

In this paper the optimal control problem is considered, when the state of the system is described by the impulsive differential equations with integral boundary conditions. By the help of the Banach contraction principle the existence and uniqueness of solution is proved for the corresponding boundary problem by the fixed admissible control. The first and the second variation of the functional is calculated. Various necessary conditions of optimality of the first and the second order are obtained by the help of the variation of the controls.

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On Stability Of Hyperbolic- Elliptic Differential Equations With Nonlocal Integral Condition

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Abstract

The nonlocal boundary value problem for a hyperbolic-elliptic equation

$$\begin{cases} u_{tt}(t) + Au(t) = f(t), \quad 0 \leq t \leq 1, \\ -u_{tt}(t) + Au(t) = g(t), \ -1 \leq t \leq 0, \\ u(-1) = \int\limits_0^1 \alpha(s)u(s)ds + \psi, \ u(0) = \varphi. \end{cases}$$

in a Hilbert space H with the self-adjoint positive definite operator A is considered. The stability estimates for the solution of this problem are established.

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FUZZY CONTINUOUS DYNAMICAL SYSTEM: A MULTIVARIATE OPTIMIZATION TECHNIQUE

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Abstract

This paper presents a multivariate optimization technique for the numerical simulation of continuous dynamical systems whose parameters, functional forms and/or initial conditions are modeled by fuzzy distributions. Fuzzy differential equation (FDE) is interpreted by using the strongly generalized differentiability concept and is shown that by this concept any FDE can be transformed to a system of ordinary differential equations (ODEs). By solving the associate ODEs one can find solutions for FDE. This approach has an inherited drawback of increasing uncertainty at each instance of time generally with nonlinear functional forms. Here we present a methodology to numerically simulate interval calculus and implements a new approach to the numerical integration of fuzzy dynamical systems, where the propagation of imprecision as a fuzzy distribution in the phase space is solved by a constrained multivariate optimization technique. Numerical simulations of some fuzzy dynamical systems (viz. Lotka Volterra model, Lorenz model) are also reported. Finally ecological degradation in wetlands of India is modeled by fuzzy initial value problem and some sustainable solution is proposed.

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Analysis of Dynamical Complex Network of Ecological Stability Diversity and Persistence

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Abstract

Explorations of ecological networks have led a long line of scientists to debate the influence of diversity (number of nodes) in terms of species richness and complexity in terms of the number and structure of interactions. This research on how vast numbers of interacting species manage to coexist in nature reveals a deep disparity between the ubiquity of complex ecosystems in nature and their mathematical improbability in theory. In this paper ecological networks are assumed to be complex dynamical network. Population dynamics is simulated over ecological complex network and species migration and changing food habits are found to be two keystones to species persistence on the earth. Also a comparative study on stability, complexity and persistence over complex dynamical network is shown. Here, we show how integrating models of food-web structure and nonlinear bioenergetic dynamics bridges this disparity and helps elucidate the mechanics of ecological complexity. Structural constraints of these networks including the trophic hierarchy, contiguity, and looping formalized by the "niche model" are shown to greatly increase persistence in complex model ecosystems. We explore the interplay of structure and nonlinear dynamics by systematically varying diversity, complexity, and function in order to "elucidate the devious strategies which make for stability in enduring natural systems." ([19]). Our exploration expands on previously proposed strategies and shows how recently discovered structural and functional properties of ecological networks appear to promote stability and persistence in large complex ecosystems.

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Analytical solution for the recovery tests after constant-discharge tests in confined aquifers

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Abstract

In this paper we provide a new analytical solution for residual drawdown during the recovery period after constant rate pumping test. We first compare the proposed solution with the existing solution, secondary we compare the solution with experimental data from field observation. The analytical solution is in perfect agreement with the experimental data for $\alpha = 0.01$ than Cooper Jacob solution. We derive a new analytical solution for determination of the skin factor without any restriction on the variables t and t'. We present an analytical solution for the drawdown response in a confined aquifer that is pumped step-wise or intermittently at different discharge rate on basis of this solution we derive an analytical solution to analyse the residual drawdown data after pumping test with step-wise or intermittently changing discharge rates.

Keywords: Recovery equations, residual drawdown, skin factor, Variable discharges

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Bright and dark soliton solutions for the variable coefficient generalizations of the KP equation

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July 12, 2012

Abstract

In this paper, by using a solitary wave ansatz in the form of sech^p and $\tanh^p$ functions, we obtain the exact bright (non-topological) and dark (topological) soliton solutions for the variable coefficient generalizations of the KP (GVCKP) equation, respectively. Note that, it is always useful and desirable to construct exact analytical solutions especially soliton-type envelope for the understanding of most nonlinear physical phenomena. The physical parameters in the soliton solutions are obtained as functions of the dependent coefficients.

Keywords: Solitons, bright and dark soliton, variable-coefficient generalizations of the KP (GVCKP) equation

PACS (2006) : 02.30 Jr, 02.70 Wz, 05.45 Yv, 94.05 Fg.

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A Characterization of Compactness in Banach Spaces with Continuous Linear Representations of the Rotation Group of a Circle.

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Abstract

Let $\mathbb{H}$ be a complex Banach space, $\mathbb{T}$ be the unit circle $\{z \in \mathbb{C} : |z| = 1\}$, $SO(2)$ be the group of all rotations of $\mathbb{T}$, $GL(\mathbb{H})$ be group of all invertible bounded linear operators on $\mathbb{H}$, $\alpha : SO(2) \to GL(\mathbb{H})$ be a continuous linear representation, $x \in \mathbb{H}$. For all $n \in \mathbb{Z}$, n-th Fourier coefficient of x with respect to the α is defined by

$$P_n(x) = \frac{1}{2\pi} \int_{\mathbb{T}} e^{-int} \alpha(t)(x) dt$$

and the Fourier series of x with respect to the α is defined by

$$\sum_{n=-\infty}^{+\infty} P_n(x). \tag{1}$$

The convergence of this series and some properties of $P_n(x)$ are investigated in [5]. In this work, a characterization of compactness in Banach space $\mathbb{H}$ is given by means of Fourier coefficients $P_n(x)$. One of the main results is as follows:

Theorem : Suppose that $dim H_n < +\infty$ for all $n \in \mathbb{Z}$. Then a closed subset $A \subset \mathbb{H}$ is compact if and only if for any $\varepsilon > 0$ there exists a natural number $N(\varepsilon)$ such that$\|\frac{n}{n+1}\sigma_n(x) - x\| < \varepsilon$ for all $x \in A$ and $n \geq N(\varepsilon)$.

Where, for all $n \in \mathbb{N} \cup \{0\}$, $\sigma_n(.) : \mathbb{H} \to \mathbb{H}$ is a linear bounded operator which is defined by

$$\sigma_n(x) = \frac{1}{n+1} \sum_{k=0}^{n} S_k(x)$$

for all $x \in \mathbb{H}$, $S_k(x)$ is the k-th partial sum of (1) for all $k \in \mathbb{N} \cup \{0\}$ and

$$H_n := \{x \in \mathbb{H} : \alpha(t)(x) = e^{int}x, \forall t \in \mathbb{T}\}$$

for all $n \in \mathbb{Z}$.

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Acknowledgement. This work was supported by the Commission of Scientific Research Projects of Karadeniz Technical University, Project number: 2010.111.3.1.

The approximate solutions of linear Goursat Problems via Homotopy Analysis Method

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Abstract

In this study we investigate the linear Goursat problems that arise in linear partial differential equations with mixed derivatives. The standart form of Goursat Problem is given by

$$\begin{aligned} u_{xt} &= f(x,t,u,u_x,u_t), \quad 0 \leqslant x \leqslant a,\ 0 \leqslant t \leqslant b, \\ u\left(x,0\right) &= g(x), \quad u(0,t) = h(t), \\ u(0,0) &= g(0) = h(0). \end{aligned}$$

The aim of this work is to present an efficient numerical procedure, namely Homotopy Analysis Method, for solving homogeneous and inhomogeneous linear Goursat problems. The reliability and efficiency of the proposed method are demonstrated by some numerical examples and performed on the computer algebraic system Mathematica 7.

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Paths of Minimal Length on Suborbital Graphs with Recurrence Relations

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Abstract

In this paper, we study suborbital graphs for congruence subgroup $\Gamma_0(N)$ of the modular group Γ to have vertices of the graph $\mathbf{F}_{u,N}$ and hyperbolic paths of minimal length with recurrence relations give rise to a special continued fraction.

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Riesz Basis Property of Eigenfunctions of One Boundary-Value Transmission Problem

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Abstract

We consider a Sturm-Liouville equation together with eigendependent boundary conditions and two supplementary transmission condition at the one inner point. Note that some special cases of the considered problem arise after an application of the method of separation of variables to the heat transfer problems, in vibrating string problems when the string is loaded additionally with point masses, in diffraction problems etc. We introduce a new inner product in the Sobolev Spaces $W_2^1(a,b)$ and show that eigenfunctions of our problem form a Riesz basis of this modified space.

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On A SUBCLASS OF UNIVALENT FUNCTIONS WITH NEGATIVE COEFFICIENTS

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Abstract

In this paper, we have introduced the subclass of univalent functions defined in the open unit disc and derived some interesting properties like coefficient estimates, distortion theorem, extreme points and radii of close- to- convexity , starlikness and convexity.

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Fine spectra of upper triangular triple-band matrices over the sequence space ℓ_p, $(0 < p < \infty)$

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Abstract The operator $A(r,s,t)$ on sequence space on ℓ_p is defined $A(r,s,t)x = (rx_{k-1} + sx_k + tx_{k+1})_{k=0}^{\infty}$ where $x = (x_k) \in \ell_p$, with $(0 < p < 1)$. The main purpose of this paper is to determine the fine spectrum with respect to the Goldberg's classification of the operator $A(r,s,t)$ defined by a triple sequential band matrix over the sequence space ℓ_p. Additionally, we give the approximate point spectrum, defect spectrum and compression spectrum of the matrix operator $A(r,s,t)$ over the space ℓ_p.

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Approximation by Certain Linear Positive Operators of Two Variables

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Abstract

In this study, we introduce positive linear positive operators which are combined the Chlodowsky and Szász type operators and study some approximation properties of these operators in the space of continuous functions of two variables on a compact set. The rate of convergence of this operators are obtained by means of the modulus of continuity. And we also obtain weighted approximation properties for these positive linear operators in a weighted space of functions of two variables and find the rate of the convergence for this operators by using weighted modulus of continuity.

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Impulsive differential equations with variable times

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Abstract

In this paper, Schauder-Tychonoff's fixed point theorem and the notion of upper and lower solutions are used to investigate the existence of solutions for first order impulsive equations.

Keywords: Impulsive equations; upper and lower solutions; fixed point.

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PARABOLIC PROBLEMS WITH PARAMETER OCCURING IN ENVIRONMENTAL ENGINEERING

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Abstract

In this work, the uniform well possedenes of singular perturbation problems for parameter dependent parabolic differential-operator equations are obtained. These problems occur in phytoremediation modelling.

Key Word: Singular perturbation, Initial value problems; Differential-operator equations; Abstract parabolic equation; Interpolation of Banach spaces; Semigroups of operators; phytoremidation modelling

AMS: 34G10, 35J25, 35J70

1. Introduction

Remediation techniques have been based on either immobilization, extraction by physick-chemical methods, landholding, or burial. These method often have some shortcoming: requiring special equipment, expensive, can remove biological activity from the soil, and can important affect the soil physical properties.

The model describing in this projet is developed in three parts. First, the dynamic portion will be developed using a a reaction-diffusion systems. Next, the cost function will involve the dynamic state variables and finally the desired EPA target will be defined as mathematical property. Assume $u_1(t,x)$, $u_2(t,x), u_3(t,x)$ are amount of heavy metal in the environment in the roots and in the shoots at t months on $x=(x_1,x_2,x_3)$ place, respectively. Since the plant toxicant interaction dynamic occurs during a harvest season, we need to describe the process one harvest cycle. The initial amount of metal in different harvest cycle depends on what is remaining in the soil at the end of the cycle. The mathematical description of this process can be obtained as the following initial value problem (IVP) for systems of delay parabolic equation with parameter

$$s\frac{\partial u_i}{\partial t}+\sum_{j=1}^{3}b_j(s,t)\,u_j(t-\sigma_j,x)=f_i(t,x)\,,\ 0<t\leq T,$$

$$u_i(t,x)=g_i(t,x)\,,\ \ -\sigma_j\leq t\leq 0,\ x\in[a,b]\,,\ i,j=1,2,3.$$

ON DARBOUX HELICES IN MINKOWSKI SPACE $\mathbb{R}_1^3$

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Abstract

In the present study, we give the conditions for a curve in the Minkowski space to be a Darboux helix. We show that α is a Darboux helix if there exists a fixed direction d in $\mathbb{R}_1^3$ such that the function $\langle W(s), d\rangle$ is constant. We give the relation between slant helice and Darboux helice. As a particular case, if we take $\|w\|$ =constant, the curves are constant precession. Some more particular cases of constant precession curves are studied.

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On the Numerical Solution of a Diffusion Equation Arising in Two-phase Fluid Flow

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Abstract

Many applied problems in fluid mechanics and mathematical biology were formulated as the mathematical model of partial differential equations. Fluid flow inside capillaries were also considered with mathematical models [1]-[3]. But it is known that due to the lack of some data and/or coefficients, many real-life problems are modeled as inverse problems [4]-[5]. In this paper, specific modeling of the fluid flow for an unknown pressure is modeled as a two-phase flow equation. The unknown pressure acting in the model can be identified by using the overdetermined condition. Difference schemes are constructed for obtaining approximate solutions of this inverse problem. Stability estimates for the solution of these difference schemes are established.

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On the Solution of a Three Dimensional Convection Diffusion Problem

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Abstract

In this paper, the Rothe difference scheme and the Adomian Decomposition method are presented for obtaining the approximate solution of three dimensional convection-diffusion problem. Stability estimates for the difference problem is presented.

Keywords: Finite difference method, Adomian Decomposition Method, Convection-diffusion equation

1 Introduction

In many important applications in engineering such as transport of air and water pollutants, convection-diffusion problems arises. An example of this kind of problem is a forced heat transfer. Several numerical methods are proposed for solving three dimensional convection diffusion problem (see [1]-[11] and the references therein). In this paper, we focus on the following mixed problem for the three dimensional convection-diffusion equation

$$\begin{cases} \frac{\partial u}{\partial t} + b_1(x,y,z)\frac{\partial u}{\partial x} + b_2(x,y,z)\frac{\partial u}{\partial y} + b_3(x,y,z)\frac{\partial u}{\partial z} \\ -\left(a_1\frac{\partial^2 u}{\partial x^2} + a_2\frac{\partial^2 u}{\partial y^2} + a_3\frac{\partial^2 u}{\partial z^2}\right) = f(t,x,y,z), \text{ in } \Omega\times P, \\ u(x,y,z,t) = 0, \text{ on } \partial\Omega\times\overline{P}, \\ u(x,y,z,0) = g(x,y,z), \text{ in } \overline{\Omega}, \end{cases} \tag{1}$$

where $\Omega = (0,1)\times(0,1)\times(0,1)$, $P = (0,T), b_1(x,y,z), b_2(x,y,z), b_3(x,y,z), g(x,y,z)$ are sufficiently smooth functions and $a_1, a_{2,}a_3$ are positive constants. Here, $b_1(x,y,z)$, $b_2(x,y,z)$, $b_3(x,y,z)$, a_1, a_2 and a_3 are velocity components of the fluid in the directions of the axes at the point (x,y,z) at time t.

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A Fuzzy Max-Min Approach to Multi Objective, Multi Echelon Closed Loop Supply Chain

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Abstract

In today's competitive markets, optimizing the process of delivering products from suppliers of raw materials to the customers for the firms formalizes an important problem in the literature. Increasingly contaminated world and limited sources of energy in recent years are regarded, it is inevitable for the mathematical models of any supply chain to have an environmentalist perspective. Hence, closed loop supply chain method has an increasing importance. In this study, a multi-objective linear model is given for the multi-echelon closed loop supply chain and the solution is obtained by utilizing Zimmermann's "min" operator with a fuzzy approach in which the minimum satisfactions of objectives are maximized. The model is to determine the locations of facilities and distribution quantity on the network regarding three objective functions, which are; minimizing time and cost, maximizing rating.

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A Fuzzy Approach to Multi Objective Multi Echelon Supply Chain

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Abstract

Recent times, companies are being forced by hard marketing conditions to make significant and strategic decisions on their supply chains. In this context, companies are trying to optimize supply chains towards customer demands and trying to prevent costs that caused by number of inactive facilities. In this study, by using AHP we make decision about potential establishment of a number of potential warehouses and distributions centers at regions to be selected from a set of possible candidates with certain possibilities of customer demands in the supply chain network of a company that is importing and exporting cleaning materials. The proposed model attempts to simultaneously minimize total cost and maximizing rating candidate locations using mixed integer linear programming. To obtain solution fuzzy decision making method is used and numerical example is illustrated.

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Modeling Voting Behavior in the Eurovision Song Contest

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Abstract

Modeling voting behavior or determinants of voting in a popular music competition such as Queen Elizabeth Piano Contest and Eurovision Song Contest have been growing tremendously after 2000s. (See [1], [4], [5], [9]). The aim of this study is also to model v_{ij}, voting behavior of juries and public opinion (via televoting system) of country $i \epsilon L$ in evaluating the singer of country $j \epsilon L$ $(i \neq j)$ where L is the total number of participants in the Eurovision Song Contest (ESC). We modeled voting behavior taking into consideration the individual characteristic of performer and voter, as well as quality of song. Characteristic properties $x_{ik},\ k = 1,2,3,\ldots,K$ of performer j and characteristics $y_{jm},\ m = 1,2,3,\ldots,M$ of voter i together affect votes given to a performer, as well as exchange of votes between two countries. Voting equation can be improved with these factors as below:

$$v_{ij} = \beta_{ij} v_{ji} + \sum_{k=1}^{K} \alpha_k x_{ik} + \sum_{m=1}^{M} \varphi_m y_{jm} + \varepsilon_{ij} \quad (1)$$

Where α_k and φ_m are parameters to be estimated. The last two parameters of right-hand side of the equation (1) are affinity and objective quality of song. These two parameters together indicate some individual characteristics of singer and voter such as gender (male, female and duet), the "language" in which song is performed (English, English +national language, French, National language), the order of "appearance" in the contest, whether the song is performed "alone" or in a "group", a dummy for "host" country (if singer represents the host country, the variable takes 1 and 0 for other), and a dummy variable to capture "cultural block" ties' effect on voting (Western, Scandinavia, Former Yugoslavia, Former Socialist and Independents). Geographic effect (neighbor_{ij}) and quality of a song are computed as below

$$\text{neighbor}_{ij} = \frac{1}{l}\sum_{j=1}^{l} v_{ij}, \qquad i = 1,2,3,\ldots,L \quad (2)$$

$$\text{quality}_{ij}{}^{g} = \frac{1}{L-l-1}\sum_{j=1}^{L-l-1} v_{ij}, \quad i = 1,2,3,\ldots,L \quad (3)$$

Estimation result of the linear voting equation 4 (including neighborhood and quality variable) shows that not only quality of the song is an important part of voting but also affinity variables are very crucial determinants of voting equation. Estimation result also indicates that order of appearance in the contest, the language of the song and the gender of the performing artist turn out to be quite important parameters in explaining voting behavior.

$$v_{ij} = \beta_{ij1} v_{ji-1} + \sum_{k=1}^{K} \alpha_{k1} x_{ik} + \sum_{m=1}^{M} \varphi_{m1} y_{jm} + \gamma_1 \text{quality}_{ij}{}^{g} + \varepsilon_{ij1} \quad (4)$$

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Derivation and numerical study of relativistic Burgers equations posed on Schwarzschild spacetime

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We consider nonlinear hyperbolic balance laws posed on a curved spacetime endowed with a volume form and identify a unique (up to normalization) hyperbolic balance law that enjoys the Lorentz invariance property also shared by the Euler equations of relativistic compressible fluids. The proposed model can be viewed as a relativistic version of Burgers equation and provides us with a simplified model on which numerical methods for hyperbolic equations can be developed and analyzed. This model is also compared with a second model derived directly from the relativistic Euler equations. We then introduce a finite volume scheme for the approximation of discontinuous solutions to the Burgerstype model when the background is chosen to be (a subset of) the Schwarzschild spacetime. Our scheme is formulated geometrically and is consistent with the natural divergence form of the balance law and applies to weak solutions containing shock waves. Most importantly, our scheme is well-balanced in the sense that it preserves static equilibrium solutions. Numerical experiments demonstrate the convergence of the proposed finite volume scheme and its relevance for computing late-time asymptotics of (possibly) discontinuous solutions on a curved background.

This presentation is based on the joint paper [2].

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Newton-Pade Approximations for Univariate and Multivariate Functions

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Abstract

The Newton-Padé approximants are a particular case of the multipoint Padé approximants, corresponding to the situation when the sets of interpolation points are nested.

One may consult papers [1-11] for the theory of those approximations for univariate functions. Recently, the authors [13] found a new form for the Newton-Padé approximations and used it in their convergence study. In [12] a multivariate generalization of the Newton-Padé approximations was introduced.

The goal of this note is two-fold. Firstly, we will give short extract from our forthcoming paper [13]. Next, we present generalizations of main lemmas for the case of multivariate functions. For the sake of simplicity we restrict ourselves to the case of two variables because the generalization to more than two variables is straightforward.

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Finite Difference Method for The Reverse Parabolic Problem with Neumann Condition

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Abstract. A finite difference method for the approximate solution of the reverse multidimensional parabolic differential equation with a multipoint boundary condition and Neumann condition is applied. Stability, almost coercive stability, and coercive stability estimates for the solution of the first and second orders of accuracy difference schemes are obtained. The theoretical statements are supported by the numerical example.

The present paper considers the multipoint nonlocal boundary value problem for then multidimensional parabolic equation with Neumann condition

$$\begin{cases} u_t(t,x)+\sum_{r=1}^{n}\left(a_r(x)u_{x_r}\right)x_r = f(x,t), \quad x=(x_1,...,x_n)\in\Omega, \quad 0\langle t\langle 1, \\ u(1,x)=\sum_{i=1}^{p}\alpha_i u(\theta_i,x)+\varphi(x), \quad x\in\overline{\Omega}, \quad 0\le\theta_1\langle\theta_2\langle...\langle\theta_P\langle 1, \\ \dfrac{\partial u(x,t)}{\partial \vec{n}}=0, \quad x\in S, \quad 0\le t\le 1 \end{cases} \tag{1}$$

under the condition

$$\sum_{k=1}^{p}\left|\alpha_k\right|\le 1.$$

Here, $a_r(x)$, $(x\in\Omega)$, $\varphi(x)$ $(x\in\overline{\Omega})$, $f(t,x)$ $(t\in(0,1),x\in\Omega)$ are given smooth functions and $a_r(x)\ge a>0$, $\Omega=(0,1)\times\cdots\times(0,1)$ is the unit open cube in the n-dimensional Euclidean space with boundary S, $\overline{\Omega}=\Omega\cup S$, and $\vec{n}$ is the normal vector to Ω.

The first and second order of accuracy in t and the second order of accuracy in space variables for the approximate solution of problem (1) are presented. The stability, almost coercive stability, and coercive stability estimates for the solution of these difference schemes are obtained. The modified Gauss elimination method for solving these difference schemes in the case of one-dimensional parabolic partial differential equations is used.

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Three Models based Fusion Approach in the Fuzzy Logic Context for the Segmentation of MR Images : A Study and an Evaluation

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Abstract

In this work we present a study and an evaluation of three models based fusion approach in the fuzzy logic context for the segmentation of MR images. The process of fusion consists of three parts : (1) information extraction, (2) information combination, and (3) decision step. Information provided by T1-weighted,T2-weighted and PD-weighted images is extracted and modeled separately in each one using FPCM (Fuzzy Possibilistic C-Means) algorithm, fuzzy maps obtained are combined with an operator of fusion which can managing the uncertainty and ambiguity in the images and the final segmented image is constructed in decision step. Some results are presented and discussed.

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Approximate Solutions of The Cauchy Problem for The Heat Equations

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Abstract

Homotopy Analysis Method (HAM) [1-2] is applied to the problem of the one-dimensional heat equations with a non-linear heat source subject to the temperature and the heat flux given at a single boundary to obtain the analytical solutions. Solutions obtained take an important place for one-dimensional heat flow as applied to a few regular geometries such as slabs, cylinders and spheres. Some of the test problems are presented to show the efficiency of HAM.

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The normal inverse Gaussian distribution: exposition and applications to modeling asset, index and foreign exchange closing prices

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Abstract

We expose the unique properties of the normal inverse Gaussian distribution (NIG) useful for modeling asset, index and foreign exchange closing prices. We further demonstrate that traditional beliefs in asset, index, and foreign exchange closing prices not being independently identically distributed random variables are fundamentally flawed. Best models are selected using a novel model selection strategy proposed by Käärik and Umbleja (2011). Our results show that closing prices of Baltika and Ekpress Grupp (companies trading on Tallinn stock exchange), FTSE100, GSPC and STI (major world indexes), CHF/JPY, USD/EUR, EUR/GBP, SAR/CHF, QAR/CHF and EGP/CHF (Foreign Exchange rates) can be modeled by NIG distribution. This means their underlying stochastic properties can fully be captured by NIG; very useful for predicting price movements, pricing models, underwriting and trading derivatives etc

Acknowledgement Research supported by Estonian Science foundation grant number 8802 and Estonian Doctoral School in Mathematics and Statistics.

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Radial Basis Functions Method for determining of unknown coefficient in parabolic equation

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Abstract

In this paper, we consider an inverse problem of finding unknown source parameter$p(t)$ and $u(x,t)$ satisfy equation

$$u_t = u_{xx} + p(t)u + f(t,x), \qquad 0 \leqslant x \leqslant 1,\ 0 < t \leqslant T, \tag{1}$$

with the initial-boundary conditions

$$u(x,0) = \varphi(x), \qquad 0 \leqslant x \leqslant 1 \tag{2}$$

$$(0,t) = \mu_1(t), \qquad 0 < t \leqslant T \tag{3}$$

$$u(1,t) = \mu_2(t), \qquad 0 < t \leqslant T \tag{4}$$

subject to the overspecification over the spatial domain

$$u(x^*,t) = E(t), \qquad 0 < x^* \leqslant 1,\ 0 < \ t \leqslant T \tag{5}$$

where $f(x,t), \varphi(x), \mu_1(t), \mu_2(t)$ and $E(t) \neq 0$ are known functions, x^* is a fixed prescribed interior point in (0,1). If $p(t)$ is known then direct initial boundary value problem $(1)-(4)$ has a unique smooth solution $u(x,t)$ [1]. If u represent a temperature distribution, then $(1)-(4)$ can be interpreted as a control problem with source parameter. Based on the idea of the radial basis functions (RBF) approximation , a fast and highly accurate meshless method is developed for solving an inverse problem with a control parameter [2]. Some numerical examples using the proposed algorithm are presented.

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Compact and Fredholm Operators on Matrix Domains of Triangles in the Space of Null Sequences

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Abstract The matrix domain X_A of an infinite matrix $A = (a_{nk})_{n,k=0}^{\infty}$ of complex numbers in a subset X of the set ω of all complex sequences is the set of all $x = (x_k)_{k=0}^{\infty} \in \omega$ for which the series $A_n x = \sum_{k=0}^{\infty} a_{nk}x_k$ converge for all n and $Ax = (A_n x)_{n=0}^{\infty} \in X$. Also, if X and Y are subsets of ω then (X, Y) denotes the set of all infinite matrices that map X into Y, that is, $A \in (X, Y)$ if and only if $X \subset Y_A$. Let c_0 denote the set sequences $x \in \omega$ that converge to zero, and $T = (t_{nk})_{n,k=0}^{\infty}$ and $\tilde{T} = (\tilde{t}_{nk})_{k,k=0}^{\infty}$ be triangles, that is, $t_{nk} = \tilde{t}_{nk} = 0$ for $k > n$ and $t_{nn} = \tilde{t}_{nn} \neq 0$ $(n = 0, 1, \ldots)$. We characterise the class $((c_0)_T, (c_0)_{\tilde{T}})$. Furthermore we obtain an explicit formula for the Hausdorff measure of noncompactness of operators L_A given by a matrix $A \in (c_0)_T, (c_0)_{\tilde{T}})$, that is, for which $L_A(x) = Ax$ for all $x \in (c_0)_T$. From this result, we obtain a characterisation the class of compact operators given by matrices in $((c_0)_T, (c_0)_{\tilde{T}})$. Finally we give a sufficient condition for an operator given by a matrix to be a Fredholm operator on $(c_0)_T$.

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Compact Operators on Spaces of Sequences of Weighted Means

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Abstract

We reduce the spaces a_0^r, a_c^r, $a_0^r(\Delta)$ and $a_c^r(\Delta)$ and simplify their dual spaces and the characterisations of matrix transformations on them in [3]. We also obtain an estimate and a formula for the Hausdorff measure of noncompactness of some matrix operators on the spaces a_0^r and a_c^r, and the corresponding characterisations of compact matrix operators.

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Extended Eigenvalues of Direct Integral of Operators

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Abstract

In this work, a connection between extended eigenvalues of direct integral of operators in the direct integral of Hilbert spaces and their coordinate operators has been investigated.

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Exponential decay and blow up of a solution for a system of nonlinear higher-order wave equations

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Abstract

This work studies a initial-boundary value problem of the weak damped nonlinear higher-order wave equations. Under suitable conditions on the initial datum, we prove that the solution decays exponentially and blows up with negative initial energy.

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A New General Inequality for double integrals

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Abstract

The inequality of Ostrowski gives us an estimate for the deviation of the values of a smooth function from its mean value. More precisely, if $f:[a,b]\to\mathbb{R}$ is a differentiable function with bounded derivative, then

$$\left| f(x)-\frac{1}{b-a}\int_a^b f(t)dt \right| \leq \left[\frac{1}{4}+\frac{(x-\frac{a+b}{2})^2}{(b-a)^2}\right](b-a)\left\|f'\right\|_\infty \tag{1}$$

for every $x\in[a,b]$. Moreover the constant $1/4$ is the best possible. Inequality (1) has wide applications in numerical analysis and in the theory of some special means; estimating error bounds for some special means, some mid-point, trapezoid and Simpson rules and quadrature rules, etc. Hence inequality (1) has attracted considerable attention and interest from mathematicans and researchers. Due to this, over the years, the interested reader is also refered to ([1]-[9]) for integral inequalities in several independent variables. In addition, the current approach of obtaining the bounds, for a particular quadrature rule, have depended on the use of peano kernel. The general approach in the past has involved the assumption of bounded derivatives of degree greater than one.

In this paper, we obtain a new general inequality involving functions of two independent variables by defining the peano kernel $K(x,y;s,t)$ as the following:

$$K(x,y;t,s)=\begin{cases}\left(t-(a+\lambda\dfrac{b-a}{6})\right)\left(s-\left(c+\lambda\dfrac{d-c}{6}\right)\right) & \text{for } a\leq t\leq x,\ c\leq s\leq y,\\ \left(t-(a+\lambda\dfrac{b-a}{6})\right)\left(s-\left(d-\lambda\dfrac{d-c}{6}\right)\right) & \text{for } a\leq t\leq x,\ y\leq s\leq d,\\ \left(t-(b-\lambda\dfrac{b-a}{6})\right)\left(s-\left(c+\lambda\dfrac{d-c}{6}\right)\right) & \text{for } x\leq t\leq b,\ c\leq s\leq y,\\ \left(t-(b-\lambda\dfrac{b-a}{6})\right)\left(s-\left(d-\lambda\dfrac{d-c}{6}\right)\right) & \text{for } x\leq t\leq b,\ y\leq s\leq d.\end{cases}$$

This inequality is a new generalization of the inequalities of Simpson and Ostrowski type obtained by Zhongxue in [9].

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Exact Solutions of the Schrödinger Equation with Position Dependent Mass for the solvable Potentials

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Abstract

In this work the infinitesimal operators of the regular representations of the group SL(2,R) are considered. According to these infinitesimal operators the Casimir operator is expressed. The Hamiltonian H is related to Casimir operator C of the group. The energy eigenvalues and the corresponding eigenfunctions are given for the solvable potentials

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Sturm Liouville Problem with Discontinuity Conditions at Several Points

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Abstract

In this paper we deal with the computation of the eigenvalues of Sturm Liouville problem with several discontinuity conditions (transmission conditions) inside a finite interval and parameter dependent boundary conditons.By using an operator theoretic interpretation we extend some classic results for regular Sturm Liouville problems.A symmetric linear operator A is defined in an appropriate Hilbert space such that the eigenvalues of such a problem coincide with those of A. Also,we obtained asymptotic formulaes for the eigenvalues and corresponding eigenfunctions.

Consider the following Sturm Liouville problem with discontinuity conditions at several points inside a finite interval,

$$\tau(u) := -u'' + q(x)u = \lambda u, \ x \in (x_0, x_1) \tag{1}$$

$$B_1(u) := \beta_1 u(x_0) + \beta_2 u'(x_0) = 0 \tag{2}$$

$$B_2(u) := \lambda\left(\alpha_1' u(x_1) - \alpha_2' u'(x_1)\right) + \alpha_1 u(x_1) - \alpha_2 u'(x_1) = 0 \tag{3}$$

$$T_k(u) := \begin{pmatrix} u(\theta_k + 0) \\ u'(\theta_k + 0) \end{pmatrix} - D_k \begin{pmatrix} u(\theta_k - 0) \\ u'(\theta_k - 0) \end{pmatrix} = 0, \quad k = \overline{1, m} \tag{4}$$

where $x_0 = \theta_0 < \theta_1 < ... < \theta_m < \theta_{m+1} = x_1$, $q \in L_2(x_0, x_1)$, λ is a complex spectral parameter. We shall assume that $\beta_1^2 + \beta_2^2 \neq 0, \alpha_1^2 + \alpha_2^2 \neq 0, \rho > 0$,where $\rho = \begin{cases} \infty & \text{, if } \alpha_1' + \alpha_2' = 0 \\ \alpha_1'\alpha_2 - \alpha_2'\alpha_1 & \text{, otherwise} \end{cases}$ and $D_k = \begin{pmatrix} \gamma_{1k} & \gamma_{2k} \\ \gamma_{3k} & \gamma_{4k} \end{pmatrix}$, $\gamma_{ik} \in \mathbb{R}$, $i = \overline{1, 4}$, $|D_k| > 0$ for $k = \overline{1, m}$. Let D_0 be the 2×2 identity matrix.

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Characterization of Three Dimensional Cellular Automata over $\mathbb{Z}_m$

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Abstract

Three dimensional cellular automata wasn't much studied by researches. Tsalides *et al.* characterized three dimensional cellular automata in [1] and then Hemmingsson investigated quasi periodic behavior of three dimensional cellular automata in [2]. In this work we study the algebraic behavior of three dimensional linear cellular automata over $\mathbb{Z}_m$. we provide necessary and sufficient conditions for a three dimensional linear cellular automata over the ring $\mathbb{Z}_m$ to be reversible or irreversible. As a consequence of our result we characterize three dimensional linear cellular automata under the null boundary conditions.

Acknowledgements: The work is supported by TÜBİTAK (Project Number: 110T713).

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Positivity of Elliptic Difference Operators and its Applications

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As is well-known that the investigation of well-posedness of various types of parabolic and elliptic differential and difference equations is based on the positivity of elliptic differential and difference operators in various Banach spaces and on the structure of the fractional spaces generated by these positive operators. An excellent survey of works in the theory of fractional spaces generated by positive multidimensional difference operators in the space and its applications to partial differential equations was given in [1]-[2]. In a number of works (see, e.g., [3]-[11], and the references therein) difference schemes were treated as operator equations in a Banach space and the investigation was based on the positivity property of the operator coefficient.

In the present paper, we consider the difference operator

$$(-1)^n \partial_{h_p}^{2n} + A_h,$$

where A_h is the self-adjoint positive definite operator in L_{2h}. Applying the method of paper [3] the positivity of this difference operator in the Holder spaces is established. In applications, the well-posedness of the Cauchy problem for parabolic differential and difference equations is investigated.

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On (α, β)-derivations in BCI-algebras

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Abstract

The notion of (regular) (α, β)-derivations of a BCI-algebra X is introduced, some useful examples are discussed, and related properties are investigated. Condition for a (α, β)-derivation to be regular is provided. The concepts of a $d_{(\alpha,\beta)}$-invariant (α, β)-derivation and α-ideal are introduced, and their relations are discussed. Finally, some results on regular (α, β)-derivations are obtained.

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Transient and Cycle Structure of Elementary Rule 150 with Reflective Boundary

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Abstract

Cellular automata are simple mathematical representation of complex dynamical systems. Therefore there are several applications of cellular automata in many areas such as coding, cryptography, VLSI design etc. [1,2]. In this study, a recurrence relation for computation minimal polynomial of transition matrix of linear elementary rule 150 with reflective boundary condition [3] was obtained. Then, the maximum transient and cycle lengths of this rule were calculated by algorithm in [4].

Acknowledgements: The work is supported by TÜBİTAK (Project Number: 110T713).

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Numerical Solution of a laminar viscous flow boundary layer equation using Haar Wavelet Quasilinearization Method

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Abstract

In this paper,we propose a wavelet method to solve the well known Blasius equation. The method is based on the Haar wavelet operational matrix defined over the interval $[0, 1]$. In this method,we have used the coordinate transformation for converting the problem on a fixed computational domain. The generalized Blasius equation arises in the various boundary layer problems of hydrodynamics and in fluid mechanics of laminar viscous flows. Comparison is made with existing solutions in literature. Haar Wavelet Quasilinearization Method is of high accuracy even in the case of a small number of grid points and without any iteration.

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Characterizations of Slant Helices According to Quaternionic Frame

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Abstract

In this study, we give the characterizations of slant helices according to quaternionic frame in 3- and 4-dimensional Euclidean spaces. Furthermore, we obtain some necessary and sufficient conditions for a space curve to be a slant helix according to quaternionic frame.

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Some Characterizations of Constant Breadth Tımelıke Curves in Minkowskı 4-space E^4

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Abstract

In this study, the differential equation characterizations of timelike curves of constant breadth are given in Minkowskı 4-space E^4. Furthermore, a criterion for a curve to be the timelike curve of constant breadth in E^4 is introduced. As an example, the obtained results are applied to the case that the curvatures k_1, k_2, k_3 and are discussed.

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Using Inverse Laplace Transform for the solution of a Flood Routing Problem

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Abstract

The inverse Laplace transform is of significant importance in mathematical sciences when an analytical solution exists in Laplace domain. So, a new solution of the linearized St. Venant equations (LSVE) has been obtained for flood routing in open channels. The LSVE has been previously used by many researchers [1] and in Laplace domain are in the matrix form

$$\frac{\partial \kappa}{\partial t} + A\frac{\partial \kappa}{\partial x} + B\kappa = 0 \tag{1}$$

where κ is transfer matrix includes deviations of discharge $q(x,t)$ and depth $y(x,t)$ around the reference values (Q_0, Y_0). In this new formulation, the Manning formula is linearized as boundary condition besides the St. Venant equations to get a Laplace transformable, simplified set of equations in Laplace domain as follows

$$\hat{q}(L,s) = k_v \hat{y}(L,s) \tag{2}$$

where $k_v = \frac{\partial Q}{\partial Y}$. A method for Laplace inversion, which provides a great convergence, very accurate response for flood routing problem is used here. As previously this method has been used for difuusion waves model [2, 3], the results show the improved De Hoog algorithm [4] provide a solution with zero error for discharge, and very small percent of error for depth. Applying the well-known Preissmann implicit scheme on the LSVE for equal condition shows that the De Hoog algorithm is in complete agreement with the numerical solution of the LSVE.

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Applied Mathematics Analysis of the Multibody Systems

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Abstract

In this work, A methodology is developed for the analysis of the multibody systems that is applied on the vehicle as a case study. The previous study is emphasized on the derivation of the multibody dynamics equations of motion for bogie [see 2]. In this work, we have developed a guide-way for the analysis of the dynamics behavior of the multibody systems for mainly validation, verification of the realistic mathematical model and partly for the design of the alternative optimum vehicle parameters .

$$\frac{\partial}{\partial t}\left[\frac{\partial E_k}{\partial \dot{p}^i}\right] - \frac{\partial E_k}{\partial p^i} + \frac{\partial E_p}{\partial p^i} + \frac{\partial E_D}{\partial p^i} = Q_i \quad (1)$$

Derivation of the DAEs

Lagrange method is used with trajectory coordinate system as seen by equation 1. to derive generalized equation of motion for the differential algebraic equations [see 4]. These generalized equations programmed in the Matlab's Symbolic Mathematics Toolbox. The size of the DAE's are 44 for the *bogie* and about 156 for the whole railway vehicle.

A methodology is developed for applied mathematics analysis of the multibody systems. This methodology can be used to compare with the symbolically derived DAEs of the motions with the previous studies for validation or the optimization of the vehicle dynamical parameters [see 1 and 3]. Case studies of the railway vehicle multibody mathematical model is tested for this methodology with a success. Although the most critical and influential symbolically varied parameter of the velocity is picked for the case study one can pick the rest of the other parameters such as mass, inertia or dimensions of the vehicle to design vehicle or mechatronic system for purposes such as stability (critical velocity for railway case) and comfort criteria.

Keywords: Computational differential-algebraic equations (CDAEs), Multibody dynamics (MBD), Eigenvalue analysis, Lagrange dynamics, Railway vehicles.

ACKNOWLEDGEMENTS

Authors would like to acknowledge the financial support of the TUBITAK with the project numbered 110M561. Authors are grateful to the BTE Department of the TUBITAK MAM Research Institute for the permition to have the real and simulated data from the TRENSIM project completed for the Turkish State Railways (TCDD) for the E43000 Locomotive Simulator.

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Multibody Railway Vehicle Dynamics Using Symbolic Mathematics

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Abstract

In this work, the Equations of Motion (EOMs) of the Multibody Dynamics is derived for a railway vehicle. The previous work of the authors is related to derive the Multibody Dynamics model of the bogie with 44 DAEs (see [1]). Lagrange dynamics is used as common approach in applied mathematics and mechanics for computational differential-algebraic equations (CDAEs). Differential equations of motions are formulized as in the generalized symbolic mathematics and applied in the Matlab's MuPad Symbolic Math Toolbox.

The size of the railway vehicle's DAEs is about 156. Finally, the results are compared using eigenvalues with previous studies in the same area with a success. The symbolic mathematics is currently used for derivation of the multibody dynamics EOMs (see [2] and [4]). Langrange dynamics for the trajectory coordinate is applied to derive generalized EOMs for the multibody dynamics. Following Equation 1 is one of the generalized equation used to derive the state space representation of the EOMs for the railway vehicle (see [3]).

$$\begin{Bmatrix} \dot{\mathbf{p}}^i \\ \ddot{\mathbf{p}}^i \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\,\mathbf{K} & -\mathbf{M}^{-1}\,\mathbf{C} \end{bmatrix} \begin{Bmatrix} \mathbf{p}^i \\ \dot{\mathbf{p}}^i \end{Bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix} \mathbf{u} + \mathbf{E} \tag{1}$$

Keywords: Computational differential-algebraic equations (CDAEs), Multibody dynamics (MBD), Eigenvalue analysis, Lagrange dynamics, Railway vehicles.

ACKNOWLEDGEMENTS

Authors would like to acknowledge the financial support of the TUBITAK with the project numbered 110M561. Authors are grateful to the BTE Department of the TUBITAK MAM Research Institute for the permition to have the real and simulated data from the TRENSIM project completed for the Turkish State Railways (TCDD) for the E43000 Locomotive Simulator.

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Existence of Global Solutions for a Multidimensional Boussinesq-Type Equation with Supercritical Initial Energy

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Abstract

In this work, global weak solutions of the multidimensional Boussinesq-type equation with power type nonlinearity $\gamma |u|^p$ and supercritical initial energy is given by potential well method. Classical energy methods can not guarantee the global existence for this type of nonlinearity. As is known the functional defined for potential well method includes only the initial displacement, and by use of sign invariance of this functional one can only prove the global existence for critical and subcritical initial energy. In the case of supercritical initial energy such a functional fails to prove the global existence. A new functional is defined, which contains not only initial displacement, but also initial velocity.

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Dissipative Extensions of Fourth Order Differential Operators in the Lim- 3 case

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Abstract

Extensions of symmetric operators arise in many areas of mathematical physics, like solvable models of quantum mechanics and quantization problems. Let us consider the scalar fourth order differential operators generated by differential expression

$$l\left(y\right)=y^{(4)}+q\left(x\right)y,\quad 0\leq x<+\infty$$

where $q\left(x\right)$ is a real continuous function in $[0,\infty)$.

In this paper, a space of boundary value is constructed for scalar fourth order differential operators in the Lim-3 case. We describe all maximal dissipative, acretive, self adjoint and other extensions in terms of boundary conditions.

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On the stability of the steady-state solutions of cell equations in a tumor growth model

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Abstract

In this study, we provide the stability analysis of the steady state solutions of endothelial, pericyte and macrophage cell equations in a mathematical model in tumor angiogenesis. We do this by studying phase plane analysis of the system of ordinary differential equations obtained from the cell equations. We also discuss the biological importance of the analysis in tumor angiogenesis.

Keywords: Angiogenesis, Phase plane analysis, Tumor cells

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The Cuttings Transport Modelling with Couette Flow

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Abstract

In the petroleum production, when an oil well is drilled, rock cuttings are transported up to the surface. As current mathematical models of the flow and transport neglect the effect of drillstring rotation, it is necessary to have a model that includes rotation effects. Predicting effective cuttings transport mechanism requires all of the parameters to be considered simultaneously. To beter understand the cuttings transport mechanism, a mechanistic model is used for cuttings in Couette and Poiseuille flow, as well as the helical flow being the superposition of Couette and Poiseuille flows.

In this paper, we present the approximate solution of cuttings transport model(Couette Flow Model) being only direction of rotation by combining Modified Differential Transform Method and Adomian Decompositon.

Couette flow velocity profile will be used in $\varepsilon\frac{d^2\mathrm{x}}{dt^2}+\frac{d\mathrm{x}}{dt}=\mathrm{V}(\mathrm{x})-\varepsilon g_0\mathrm{j}$ model equation instead of V(x), where V(x) is fluid velocity at x location, μ_f is dynamic viscosity, a_p is particle size and $k=6\pi a_p\mu_f$ (Kurzweg, 1995). Nondimensional parameters are $\varepsilon=m\omega/k=O(10^{-1})$ and $g_0=g/\omega^2 r_0=O(10^{-1})$.

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On the Numerical Solution of Diffusion Problem with Singular Source Terms

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Abstract.
Partial differential equations with singular (point) source terms arise in many different scientific and engineering applications. Singular means that within the spatial domain the source is defined by a Dirac delta function. Our interest is this type of problems is motivated by mathematical modelling of forecasting and development of new gas reservoirs [1, 2, 3, 4].

Solutions of the problems having singular source terms generally have lack of smoothness, which is an obstacle for standard numerical methods. Therefore, solving these type of problems numerically requires a great deal of attention [5, 6, 7].

In this paper we discuss the numerical solution of initial-boundary value problem with singular source terms

$$\begin{cases} u_t = D\,u_{xx} + k_1\delta(x-a_1) + k_2\delta(x-a_2), \quad 0<x<1, \quad t>0, \quad 0<a_1<a_2<1, \\ u(t,0)=u_L, \quad u(t,1)=u_R, \quad t\ge 0, \\ u(0,x)=\varphi(x), \quad 0\le x\le 1, \end{cases} \tag{1}$$

where $\delta(x)$ is a Dirac delta function. We follow the standard finite volume approach based on the integral form of (1). We consider this approach more natural than the finite difference one directly based on the differential form, since for the integral form the treatment of the Dirac delta function expression is mathematically clear. For ease of presentation, we assume that there are only two source terms. The presented material is extendable to the case with more than two source terms. Finally, this study can be readily extended to the case with time-dependent source terms.

Keywords: Diffusion Equation; Singular Source Terms; Finite Volume Method
PACS: 02.30.Jr, 02.60.Cb, 02.60.Lj, 87.10.Ed

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One Boundary-Value Problem Perturbed by Abstract Linear Operator

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Abstract

The investigation of regular boundary value problems for which the eigenvalue parameter appears in both the ordinary differential equation and the boundary conditions originates from the Birkhoff's work [3]. In recent years, more and more researchers are interested in the discontinuous Sturm-Liouville problems. Various physics applications of this kind of problems are found in many literatures (see [1], [2], [6]). The purpose of this paper is to study a Sturm-Liouville problem with discontinuities in the case when an eigenparameter appears not only in the differential equation but also in the boundary conditions. Morever, the "differential equation" contained also an abstract linear operator (unbounded in general) in the Hilbert space $L_2(-1,0) \oplus L_2(0,1)$. We apply a different approach for investigation some spectral properties of this problem.

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Using expanding method of (G'/G) to find the travelling wave solutions of nonlinear partial differential equations and solving *mkdv* equation by this method

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Abstract

Expanding method of (G'/G) can be implemented to find survey solutions of travelling wave of some nonlinear partial differential equations. The answers depend on hyperbolic functions, trigonometric functions and rational functions (see [1]).

This method, converts nonlinear partial differential equations into a plane differential equation. It is possible to use this method to solve integrable equations and non-integrable equations. In this paper, by describing the method we analysis the application of it to solve *mkdv* equation (see [2-3]).

Phenomenon in physics and other fields are often described by nonlinear partial differential equations. During 40 years ago, finding survey solutions of nonlinear partial differential equations by implementing different methods have been the target of many researchers and the powerful methods of inverse diffusion method, homogeneous equilibrium,.expanding method of (G'/G) are proposed that are based on assumptions that the solutions of travelling wave of nonlinear partial differential equations can be expressed in (G'/G) by polynomial and $G = G(\xi)$is correct in second order linear ordinary differential equations $(LODE)$. The degree of polynomial can be obtained by balance between the highest-order derivative of the dependent variable in linear part of the differential equation with highest-order of dependent variable in nonlinear part that is appeared in ODE.

Definitions and Basic preliminaries:

1.Balance number

balance number of m can be obtained by balance between the highest-order derivative of the dependent variable in linear part of the differential equation with highest-order of dependent variable in nonlinear part that is appeared in ODE.

2.Explaining of (G'/G) Expanding Method

We consider nonlinear differential with independent variable of x and t

$$P(u, u_t, u_x, u_t t, u_x t, u_x x, ...) = 0 \tag{1}$$

Which $u = u(x, t)$ is an unknown function, P is a polynomial in $u = u(x, t)$ and has been its various partial derivative that include higher order derivative and nonlinear parts.

3.Solving* mkdv *Method by (G'/G) Expanding Method

In this section, we consider *mkdv* as the following

$$u_t - u^2 u_x + \delta u_{xxx} = 0 \qquad \delta > 0 \tag{2}$$

We intend to find the solution of above traveling wave equation

$$u(x, t) = u(\xi) \qquad \xi = x - vt \tag{3}$$

The speed of V will be determined later.

In this paper, (G'/G) Expanding Method proposed by Wang, is used to solve *mkdv* method. It is clear that solving nonlinear partial differential equations needs suitable change of variable and after solving this kind of equation, we reach a solution. As we observed, by using (G'/G) Expanding Method, it is possible to solve these equations and have more solutions without considering specific change of variables. This method has various applications; as it is a direct and survey method to find travelling wave solutions of nonlinear partial differential equations and the outcome results can affect the future researches significantly.

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Weighted Bernstein Inequality for Trigonometric Polynomials on a Part of The Period

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Abstract

In this study we give a weighted Bernstein inequality for trigonometric polynomials on a part of the period.

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MIXED PROBLEM FOR A DIFFERENTIAL EQUATION WITH INVOLUTION UNDER BOUNDARY CONDITIONS OF GENERAL FORM

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Abstract

To solve the mixed problem for a partial differential equation with involution and a symmetric potential there was found an explicit analytical representation by the Fourier method. The problem was considered under general boundary conditions with constant coefficients by a space variable. At the same we used the methods for avoiding the termwise differentiation of a functional series and applying the minimal conditions on initial data of the problem.

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An Application on Suborbital Graphs

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Abstract

In this paper, we investigate suborbital graphs for the action of the normalizer of $\Gamma_0(\mathrm{N})$ in $\mathrm{PSL}(2,\mathbb{R})$, where N will be of the form 2^8p^2 , $p > 3$ and p is a prime. In addition we give the conditions to be a forest for normalizer in the suborbital graph $F\left(\infty, \frac{u}{2^8p^2}\right)$.

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Cellular Automata Based Byte Error Correcting Codes over Finite Fields

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Abstract

Reed-Solomon codes are very convenient for burst error correcting codes, but as the number of errors increase, the circuit structure for Reed-Solomon codes become very complex. The modular and regular structure of cellular automata can be constructed with VLSI economically. Therefore, in recent years, cellular automata have became an important tool for error correcting codes. For the first time cellular automata based byte error correcting codes analogous to extended Reed-Solomon codes over binary fields was studied by Chowdhury *et al.* in [1] and Bhaumik *et al.* improved that coding-decoding scheme in [2]. In this study cellular automata based double-byte error correcting codes are generalized from binary fields to primitive finite fields $\mathbb{Z}_p$.

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On The First Fundamental Theorem for Special Dual Orthogonal Group $SO(2, D)$ And its Application to Dual Bezier Curves

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Abstract

Let D be set of dual numbers. In this work we study the first fundamental theorem for special dual orthogonal transformations group $SO(n, D)$ in case of $n = 2$. Then our getting results compared the special orthogonal transformations group $SO(4, R)$ in R^4 because D^2 is isomorph to R^4 So we showed that the minimal conditions of the dual vectors are more less than minimal conditions of real vectors.

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On Euler's differential method for continued fractions

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Abstract

A continued fraction is an expression of the form

$$a_0 + K_{k=1}^{\infty}\left[\frac{b_k}{a_k}\right] = a_0 + \cfrac{b_1}{a_1 + \cfrac{b_2}{a_2 + \cfrac{b_3}{a_3 + \cdots}}},$$

where $a_0, a_1, a_2, \dots$ and $b_1, b_2, b_3, \dots$ are two sequences of real or complex numbers. It is remarkable that rational and irrational numbers can be clearly distinguished by continued fractions. In the 19th century, theory of continued fractions was one of the most popular areas of investigation in mathematics. The great mathematicians such as Karl Jacobi, Oscar Perron, Charles Hermit, Karl Friderich Gauss, Augustin Cauchy, Thomas Stieltjes etc. have contributed to the theory [1].

We study the continued fraction $f(t) = K_{k=1}^{\infty}\left[\frac{k+t}{k}\right]$, which depends on the parameter $-1 < t < \infty$. This continued fraction was studied by Euler. Using the Euler's differential method, which was not used by mathematicians for a long time, we derive a new formula

$$f(t) = \frac{\int_0^1 (1-x)^{p-t} \frac{d^p}{dx^p}(x^{t+1}e^x)\, dx}{\int_0^1 (1-x)^{p-t} \frac{d^p}{dx^p}(x^t e^x)\, dx},\ p-1 < t \le p+1,$$

where $p = 0, 1, 2, \dots$. For the integer values $t = p = 1, 2, \dots,$

$$f(p) = (p+1)\frac{\sum_{k=0}^{p-1} \frac{a_{p,k}}{p-k+1}}{\sum_{k=0}^{p-1} a_{p,k}},$$

where

$$a_{p,k} = \begin{pmatrix} p \\ k \end{pmatrix} \cdot \frac{1}{(p-k-1)!}.$$

Previously, it was proved by Euler that $f(0) = (e-1)^{-1}$. Using numerical methods it is found that the function $\sigma(t) = \sqrt{t} - f(t)$ is slowly increasing and $\lim_{t\to\infty} \sigma(t) = 0.25$.

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Almost Convergence and Generalized Weighted

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Abstract

In this paper, we investigate some new sequence spaces which naturally emerge from the concepts of almost convergence and generalized weighted mean. The object of this paper is to introduce to the new sequence spaces obtained as the matrix domain of generalized weighted mean in the spaces of almost null and almost convergent sequences. Furthermore, the beta and gamma dual spaces of the new spaces are determined and some classes of matrix transformations are characterized.

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Wavelet-based prediction of crude oil prices

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Abstract

There are two kinds of transactions in the crude oil markets; one is based on immediate delivery while the other one on future delivery. The spot market is dependent on the first kind of transactions and the future market is associated to the second one. Market condition (e.g. market risk, irrational trading, etc.) along with other factors (e.g. credit risk, insurance risk, seasonal factors and etc.) is often the main cause of uncertainty in the crude oil markets. Therefore, the future markets (leading markets) are built up to provide a cover structure for these uncertainties. Also crude oil future contracts, determine definitive prices in future deadlines to buy or sell according to specific criteria of delivery and payment. On the other hand, future prices reflects the markets expectations about future conditions. Consequently, large differences between futures and spot prices is often used to describe the overall market conditions. Wavelets are used as a legitimate alternative alternative for irregular situations such as data or signals with scaled features, or containing discontinuities and sharp edges and so on (see [1-2]).

In this study, we are going to use the wavelets as a suitable tool to investigate its performance in the crude oil futures markets (see [3]). We intend to provide forecasts over different forecasting horizons by introducing a prediction procedure and predicting future prices based on the wavelets by utilizing a series of data from the crude oil market and at last putting the results in comparison with the crude oil future markets data. **Definitions and Basic preliminaries**

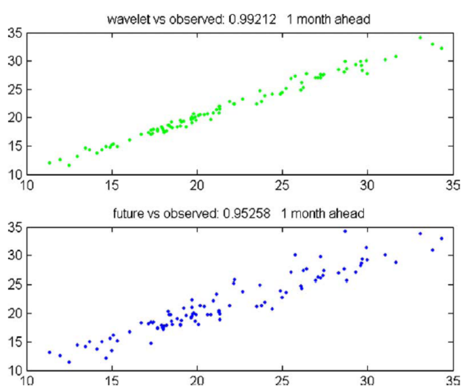
1.Multi-scale analysis

multi-scale analysis with a sequence of involute sub-space V_j of functional space of procedure V with null common point and at dense in $L_2(R)$. This analysis is a discretion at different levels of scalability, which requires two-scale relationship such as $f(x) \in V_j \iff f(2x) \in V_{j-1}$ (see [4-5]).

2.discrete wavelet transform(DWT)

discrete wavelet transformation enables us to discrete a time based sequence to subsequences with different scales in order to extract important hidden information and unstable features (see [4-5]).

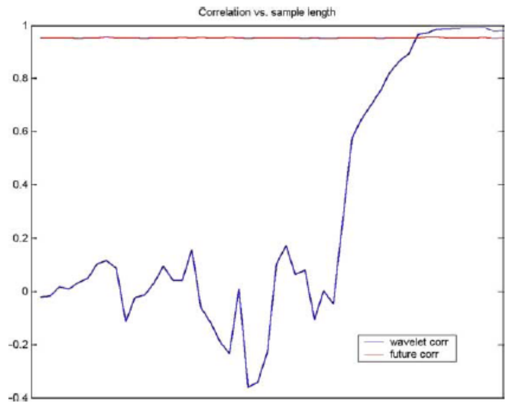
We present a procedure to predict crude oil prices for time series of 1, 2, 3 and 4 month and then compare the predicted values with actual expected prices of future market in mentioned time series and as for 1 month time series the result are shown in below figure:



Forecast results in contrast with observed values

Forecasting horizon	Wavelet-based forecast	Futures
1 month ahead	0.992	0.952
2 months ahead	0.998	0.903
3 months ahead	0.995	0.841
4 months ahead	0.998	0.772

And as you can see in below figure, wavelet based prediction procedure is more efficient for sample with a value bigger than 100 .



Applicable procedure will be created by some main key properties of wavelets and is established based on discrete wavelet transformation (DWT) on Average monthly time series of crude oil. Wavelet based prediction procedure which is used in this study, can be applied to examination of the dynamic properties of various financial and economical phenomenon, like economic time series. also predicted crude oil prices based on wavelets can be used to determine oil prices in future contracts.

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Numerical solution of a time-fractional Navier–Stokes Equation with modified Riemann-Liouville derivative

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Abstract

In this paper, fractional variational iteration method (FVIM) is implemented to give an approximate analytical solution of a time-fractional Navier–Stokes Equation. Fractional derivatives are described in the Riemann-Liouville derivative. A new application of fractional variational iteration method (FVIM) was extended to derive analytical solutions in the form of a series for these equations. By using an initial value, the explicit solution of the equation has been presented in the closed form and then its numerical solution has been showed graphically.The behavior of the solutions and the effects of different values of fractional order α are indicated graphically. The results obtained by the FVIM reveal that the method is performs extremely well in terms of efficiency and simplicity method for nonlinear differential equations with modified Riemann-Liouville derivative.

Keywords: Fractional variational iteration method, A time-fractional Navier–Stokes Equation, Riemann-Liouville derivative, Fractional calculus

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The Modified Simple Equation Method for Solving Some Nonlinear Evolution Equations

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Abstract

In this paper we applied modified simple equation method (MSEM) for solving some nonlinear evolution equations which are very important in applied sciences.

We consider a nonlinear evolution equation:

$$F\left(u, u_t, u_x, u_{xx}, \ldots\right) = 0 \tag{1}$$

where F is a polynomial in u and its partial derivatives.

Step 1. Using the wave transformation

$$u = u(\xi),\ \xi = x - t \tag{2}$$

From (1) and (2) we have the following ODE:

$$P\left(u, u', u'', u''', \ldots\right) = 0 \tag{3}$$

where P is a polynomial in u and its total derivatives and $' = \frac{d}{d\xi}$.

Step 2. We suppose that Eq. (3) has the formal solution:

$$u(\xi) = \sum_{k=0}^{N} A_k \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^k \tag{4}$$

where A_k are arbitrary constants to be determined such that $A_N \neq 0$ while $\psi(\xi)$ is an unknown function to be determined later.

Step 3. We determine the positive integer N in (4) by balancing the highest order derivatives and the nonlinear terms in Eq. (3).

Step 4. We substitute (4) into (3), we calculate all the necessary derivatives $u', u'', \ldots$ and then we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\frac{\psi'(\xi)}{\psi(\xi)}$ and its derivatives. In this polynomial, we equate with zero all the coefficients of it. This operation yields a system of equations which can be solved to find A_k and $\psi(\xi)$. Consequently, we can get the exact solution of Eq. (1).

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Application of Cross Efficiency in Stock Exchange

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Abstract

This paper firstly revisits the cross efficiency evaluation method which is an extension tool of data (envelopment analysis. In this paper, we consider the $DMUs$ as the players (institutions) in a cooperative game, where the characteristic function values of institutions are defined to compute the Shapley value of each DMU (institution), and the common weights associate with the imputation of the Shapley values are used to determine the ultimate cross efficiency scores for institution of Stock Exchange of Tehran. This paper introduces the models for computing benefit for each institution. Using shapely value we obtain the effect of each institution, and through determining common weight for each company we find out the ultimate weight which shows how much the existence or not existence of that institution affects the interesting competence.

Keywords: Data Envelopment Analysis (DEA), Cross efficiency, Cooperative game, Shapley value, Common weights, stock exchange.

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Application of the Trial Equation Method for some Nonlinear Evolution Equations

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Abstract

Nonlinear partial differential equations have important applications in physics, engineering and applied mathematics. Mathematical modelling of physics and engineering problems usually results in nonlinear partial differential equations. To find the travelling wave solutions of nonlinear evolution equations several methods [1-6] have been proposed. This study presents an application of the trial equation method for nonlinear partial differential equations. The trial equation method is used to obtain exact travelling wave solutions of some nonlinear evolution equations arising in mathematical physics.

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Paranormality of Some Class of Differential Operators for First Order

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Abstract

In this work, the paranormality properties of some class direct sum of differential operators for first order in the Hilbert space of vector-functions in the finite interval are investigated. Finally, a spectrum of these operators is researched.

Keywords: Selfadjoint and Paranormal Operator; Direct Sum of Operators and Hilbert Spaces; Spectrum.

2000 AMS Classification: 47A20, 47A10

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Oscillation Theorems for Second-Order Damped Dynamic Equation on Time Scales

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Abstract

Much recent attention has been given to dynamic equations on time scales, or measure chains, and we refer the reader to the landmark paper of S. Hilger [1] for a comprehensive treatment of the subject. A book on the subject of time scales by Bohner and Peterson [2] also summarizes and organizes much of the time scale calculus.

In this paper we shall study the oscillations of the following nonlinear second-order dynamic equations with damping

$$(r(t)\Psi(x^{\Delta}(t))^{\Delta} + p(t)\Psi(x^{\Delta}(t)) + q(t)f(x^{\sigma}(t)) = 0,\ t \in \mathbb{T}, \tag{1}$$

where $\Psi(t)$, $f(t)$, $p(t), q(t)$ and $r(t)$ are rd-continuous functions. By using a generalized Riccati transformation and integral averaging technique, we establish some new sufficient conditions which ensure that every solution of this equation oscillates. Throughout this paper, we will assume the following hypotheses:

(H_1) $p(t),\ q(t) \in C_{rd}(\mathbb{R}, \mathbb{R}^+)$,

(H_2) $\Psi : \mathbb{T} \to \mathbb{R}$ is such that $\Psi^2(u) \leq \kappa u \Psi(u)$ for $\kappa > 0$, $u \neq 0$,

(H_3) $f : \mathbb{R} \to \mathbb{R}$ is such that $\frac{f(u)}{u} \geq \lambda > 0$, and $uf(u) > 0,\ u \neq 0$,

(H_4) $r(t) \in C^1_{rd}([t_0, \infty), \mathbb{R}^+),\ \int_{t_0}^{\infty} (\frac{1}{r(t)} e_{\frac{-p(t)}{r(t)}}(t, t_0))\Delta t = \infty$.

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This work was supported by Research Fund of the Erciyes University. Project Number:FBA-11-3391

On the fine spectrum of the Λ operator defined by a lambda matrix over the sequence space c_{0} and c

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Abstract

The main purpose of this paper is to determine the fine spectrum with respect to the Goldberg's classification of the operator Λ defined by a lambda matrix over the sequence spaces c_{0} and c. As a new development, we give the approximate point spectrum, defect spectrum and compression spectrum of the matrix operator Λ on the sequence spaces c_{0} and c.

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On Hadamard Type Integral Inequalities For Nonconvex Functions

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Abstract

Convexity plays a central and fundamental role in mathematical finance, economics, engineering, management sciences and optimizastion theory. In recent years, several extensions and generalizations have been considered for classical convexity. A significant generalization of convex functions is that of φ-convex functions introduced by Noor in [3]. In [3] and [6], the authors have studied the basic properties of the φ-convex functions. It is well-know that the φ-convex functions and φ-sets may not be convex functions and convex sets. This class of nonconvex functions include the classical convex functions and its various classes as special cases. For some recent results related to this nonconvex functions, see the papers [3]-[6]. In this article, using functions whose derivatives absolute values are φ-convex and quasi-φ-convex, we obtained new inequalities releted to the right and left side of Hermite-Hadamard inequality. In particular if $\varphi = 0$ is taken as, our results obtained reduce to the Hermite-Hadamard type inequality for classical convex functions.

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A geometrical approach of an optimal control problem governed by EDO

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Abstract:

The theory of optimal control is a very important branch of optimization, the resolution of the problems controls optimal asks for the intervention of several mathematical tools, in particular the partial derivative equations. In this work one gives a geometrical approach of a problem of optimal control, it where one calls on the basic notions of the calculation of the variations such as the equation of Euler-Lagrange which is a requirement of optimality, the principle of maximum of Pontriagaine (PMP), which gives an analytical aspect to the problem controls optimal and makes it possible to study unquestionable property of the functions which defines the criterion to be minimized, the regularity of the solutions (minimum or maximum). An other very important aspect is well geometrical aspect which is used to find the geodetic ones, their natures, their numbers which requires a geometrical luggage such as the fields, of vector, the vector spaces, the curve acceptable… Then can about it defines a problem controls optimal controlled by EDO geometrically by giving some conditions.

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Existence of Local Solution for a Double Dispersive Bad Boussinesq-Type Equation

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Abstract

In this work, we consider a purely spatial higher order bad Boussinesq-type equation. We obtain the existence and uniqueness of the local solutions. The local existence of the solution is given by aid of contraction mapping principle.

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A Perturbation Solution Procedure for a Boundary Layer Problem

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Abstract

A perturbation algorithm using a new transformation is introduced for boundary-value problem with small parameter multiplying the derivative terms. To account for the linear and the nonlinear dependence of the function, we exhibit the function f for the system. We introduce the transformation $T_e = f(x,\gamma;\varepsilon)\cdot x$, where f depends on x,γ and ε. Results of Multiple Scales method, method of matched asymptotic expansions and our method are contrasted.

We consider the following boundary-value problem

$$\begin{aligned} &\varepsilon y'' + y' + y^2 = 0 \\ &y(0) = 0 \qquad y(1) = \frac{1}{2} \end{aligned} \tag{1}$$

Where ε is a small dimesionless positive number. It is assumed that the equation and boundary conditions have been made dimensionless.

In Direct Perturbation Method, secular terms appear of higher orders of the expansion invalidating the solution. In order the avoid this problem a new transformation has been proposed in our study.

The new transformation is defined as,

$$T_e = f(x,\gamma;\varepsilon)\cdot x \tag{2}$$

Using the chain rule, we transform the derivate accordingly

$$\begin{aligned} &\frac{dy}{dx} = \frac{dy}{dT_e}\frac{dT_e}{dx} = \frac{dy}{dT_e}\left(\frac{df}{dx}x + f\right) = y'(f_x x + f) \\ &\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dT_e}\left(\frac{df}{dx}x + f\right)\right) = \left[\frac{d}{dT_e}\left(\frac{dy}{dT_e}\right)\right]\left(\frac{df}{dx}x + f\right)\frac{dT_e}{dx} + \frac{dy}{dT_e}\left[\frac{d}{dx}\left(\frac{df}{dx}x + f\right)\right] \\ &= \frac{d^2y}{dT_e^2}\left(\frac{df}{dx}x + f\right)^2 + \frac{dy}{dT_e}\left(\frac{d^2f}{dx^2}x + 2\frac{df}{dx}\right) = y''(f_x x + f)^2 + y'(f_{xx}x + 2f_x) \end{aligned} \tag{3}$$

So, we have obtained a more effective parameter expression T_e without losing the original parameter x,γ and ε. Thus, speeding up and slowing down control of the time parameter will be available as in Method of Multiple Scales.

In Equation (3) first order derivatives according to new variable T_e appear in the second order derivative expressions according to original time variable x. So, we are able to have information about parameters of nonlinear differential equation and to interpret the results.

By this new transformation we have the advantages of both method of matched asymptotic expansions and Method of Multi Scales [1-6].

Using perturbation algorithm with new transformation, a more effective time expression without losing the original time parameter t have been obtained. Information about parameters of nonlinear differential equation and interpretation of the results has been achieved. Applying this transformation on boundary value problem the results obtained are compared with the results of the studies conducted to time.

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Solution of Differential Equations by Perturbation Technique Using any Time Transformation

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Abstract

A perturbation algorithm using any time transformation is introduced. To account for the nonlinear dependence of the function, we exhibit the function *f* of the system in the differential equation. To this end, we introduce the transformation $T_e = f(w,t)\cdot t$, where f is a function that depends on *t* or *w*. The problems are solved with new time transformation: Linear damped vibration equation, classical Duffing equation and damped cubic nonlinear equation. Results of Multiple Scales, Lindstedt Poincare method, new method and numerical solutions are contrasted [1-6].

Solution of Differential equations by perturbation technique using any time transformation. In Direct Perturbation Method, mostly secular terms appear of higher orders of the expansion invalidating the solution. In order the avoid this problem a new time transformation has been proposed in our study.

The new time transformation is defined as,

$$T_e = f(w,t)\cdot t \tag{1}$$

Using the chain rule, we transform the derivate accordingly

$$\begin{aligned} \frac{du}{dt} &= u'(\dot{f}(w,t)t + f(w,t)) \\ \frac{d^2u}{dt^2} &= u''(\dot{f}(w,t)t + f(w,t))^2 + u'(\ddot{f}(w,t)t + 2\dot{f}(w,t)) \end{aligned} \tag{2}$$

So we have obtained a more effective time expression T_e without losing the original time parameter *t* using the function *f*. Thus, speeding up and slowing down control of the time parameter will be available as in Method of Multiple Scales.

In Equation (2) first order time-derivatives according to new time variable T_e appear in second order time derivative expressions according to original time variable. So, we are able to have information about some parameters of nonlinear differential equation, and to interpret the results.

By this new time transformation we have the advantages of both Lindstedtpoincare method and Method Multi Scales .

Using a new perturbation algorithm with new time transformation, we showed that, first we have obtained a more effective time expression without losing the original time parameter *t* using the function *f*. We are able to have information about some parameters of nonlinear differential equation, and to interpret the results. When we apply this transformation on the known Duffing equation with the results of the studies conducted to date have compared the results obtained.

We found in this new time with the transformation of the solutions are compared with approximate solutions do not differ in Results of Multiple Scales, Lindstedt Poincare method and we found that the approximate solutions.

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Aproximation Properties of a Generalization of Linear Positive Operators in C[0,A]

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Abstract

In this paper we study the order of convergence of a generalization of positive operators by means of the functions from Lipschitz class. We use the test functions $\left(\frac{x}{1+x}\right)^{\upsilon}$ for $\upsilon = 0, 1, 2$, a Korovkin type theorem given by [1]. Furthermore we estimate the rate of convergence of these operators.Some figures correspond to obtaining results are given. Finally, the algorithm used in the program has been added.

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Three-term Asymptotic Expansion for the Moments of the Ergodic Distribution of a Renewal-reward Process with Gamma Distributed Interference of Chance

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Abstract

In this study, a renewal-reward process with a discrete interference of chance $(X(t))$ is investigated. We assume that $(X_\lambda(t))_{t\geq 0}$ is a renewal-reward process with a gamma distributed interference of chance with parameters (α,λ). Under the assumption that the process is ergodic, the paper provides the three-term asymptotic expansions for the moments EX_λ^n, $n\in\mathbb{N}$, as $\lambda\to 0$.

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Blow up of a solution for a system of nonlinear higher-order wave equations with strong damping

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Abstract

This work studies a initial-boundary value problem of the strong damped nonlinear higher-order wave equations. Under suitable conditions on the initial datum, we prove that the blow up of the solution.

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Reduction of spectral problem of Cauchy-Riemann operator with homogeneous boundary conditions to an integral equation

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Abstract

In this paper the problem on the eigenvalues of the Cauchy-Riemann operator with homogeneous boundary conditions is reduced to an integral equation In the functional space $C(|z| \leq 1)$ we consider the operators generated by differential operation of the Cauchy-Riemann

$$K\omega\left(z\right) = \frac{\partial\omega\left(z\right)}{\partial\overline{z}},$$

where $z = x + iy, \overline{z} = x - iy, \frac{\partial}{\partial\overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$ on the set

$$D\left(K\right) \subset \left\{\omega\left(z\right) \in C(|z| \leq 1), \frac{\partial\omega\left(z\right)}{\partial\overline{z}} \in C(|z| \leq 1)\right\}.$$

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A Note on Some Elementary Geometric Inequalities

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Abstract

In present paper, solutions of some elementary geometric inequalities are obtained. Firstly, we get a more useful inequality by specifying the largest lower and smallest upper bounds, to be able to end the inaccuracy of the following inequality. Let a, b, c are lengths of sides of a triangle, and if distances of a taking point in the inner region of a triangle to the vertices are x, y, z, then following inequality satisfies

$$\frac{1}{2}\left(a+b+c\right) < x+y+z < a+b+c.$$

Nevertheless it is true and useful, but it has not accurate boundaries. Because neither $\frac{1}{2}\left(a+b+c\right)$ is the largest lower bound, nor $a+b+c$ is the smallest upper bound of this sum. But in university preparation course books and textbooks, which describe this inequality, they resolved by accepted these greatest lower bound and least upper bound, so that incorrect results were obtained. In this work, we solved this inequality with more useful bounds. Namely,

1. *If distances of a taking point in the inner region of a triangle to the vertices are x, y, z which have lengths of sides a, b, c, and the area A, and also measures of all internal angles are smaller than 120 degrees, then following inequality satisfies*

$$\sqrt{\frac{1}{2}\left(a^2+b^2+c^2+4\sqrt{3}A\right)} \leq x+y+z < \max\left\{a+b, a+c, b+c\right\}.$$

2. *If distances of a taking point in the inner region of a triangle to the vertices are x, y, z which have lengths of sides a, b, c, and the area A, and one of the measure of internal angle is greater than or equal to 120 degrees, then following inequality satisfies*

$$\min\left\{a+b, a+c, b+c\right\} < x+y+z < \max\left\{a+b, a+c, b+c\right\}.$$

We have stated and proved some theorems and lemmas that have done before (see in [1-5]). One of the our theorems is famous one, namely the Toricelli-Fermat point that was solved in many ways.
In [6] and [7], we observed that Mustafa Yağcı have studied such as this work nicely. But, our proofs are completely different and some parts are more simple and clearer. In [7], he gave a problem. He claimed the problem he has given can not be solved using only geometry, but calculus is also used to solve the problem. We decided to prove this problem given in [7]. After showing the existence and uniqueness of the triangle $\triangle XYZ$ defined in the problem, we tried to solve the problem by using only geometrical way and we succeeded. The most interesting part of our article is second part, proving this following problem given below:

Problem. *For a given point P, on changing plane of the X, Y and Z let be* $|PX| = a$, $|PY| = b$, and $|PZ| = c$. *Then the circumference of the triangle $\triangle XYZ$ has the largest value, when P is at the inner center of the triangle. Secondly, what can be the minimum value of the sum of a, b and c?*

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Solving Crossmatching Puzzles Using Multi-Layer Genetic Algorithms

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Abstract

Nowadays, puzzles are used commonly as a fountain head of our monotonous lives or to spend free time. Crossmatching puzzle (CMP) is quite similar to the crossword puzzle (CWP). There are detection key table and control key table in crossmatching puzzles instead of questions in crosswrod puzzle which are written from left to right and from top to bottom. Letters in each row of the main solution table are arranged in an order in detection key table. In the same way, letters in the main solution table are arranged in order column by column are put in the control key table. Therefore, the main table is tried to solve with crossing of the letters in the detection key table and control key table.

For solution of cross-matching puzzle can be used depth search algorithm. However, in spite of depth search algorithm gives the exact result, size of puzzle as augments computing time of solution increases exponentially and it makes the solution of puzzle impossible. In this case, stochastic search is better to use instead of deterministic search algorithm.

In this study, improved genetic algorithm as multi-layer is used as stochastic search method [2]. In this algorithm, each letters represent a gene and each rows represent a chromosome. An individual is generated by chromosomes as number of rows come together. Fitting function of created individual is determined according to fitness of control key table.

Keywords: *Crossmatching Puzzle, Multi-Layer Genetic Algorithm*

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Generate Adaptive Quasi-Random Numbers

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Abstract

Random number generation, especially with the development of computer technology has an important place in the world of it.Uniform distribution random number generation can be done in almost all programming languages. Also in other distributions number generation can be produced with the help of the generated number from uniform distribution [1]. Games, education and simulation such as applications programs which frequently used random number generation, is produced in the form of discrete data. Sometimes random numbers generation produced with the same values observe consecutive. In most applications (such as education) to purify this effect we can reused the resulting number again. But it reduces the amount of numbers, the random number will be in facilitates prediction.

Indeed, as it reduced the probability of the taken number at the same time for making the prediction of the next number difficult,variable produced random numbers with their probability [2].

The frequency of region of each generated random number (f_i) are stored in an increase. The next random number which be generated,if it selected in each region,that number is considered to be the number. In this method,because of the equal intervals of the numbers they have equal probability.The aim here is to reduce the probability of selecting the same number. In this case,the interval between the numbers must be narrowed. The process of increasing of the propbability of narrowering the interval transferred to other numbers. In this case, interval's limits will be changed.

In statistical simulations,deviations from randomness is be reduced by taking too much random numbers. In this work, reduction of the amount of deviation,a set of data as a result of simulation provides an approach theoretical curve more rapidly. Thus simulation results can be reached with less data. This provides a reduction of the computing time.

Keywords: *Adaptive Quasi Random Number, Statistical Simulation*

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Polygonal Approximation of Digital Curve Using Artificial Bee Colony Optimization Algorithms

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Abstract

In image processing and pattern recognition, it is an important concept defining two-dimensional objects in the image [1]. Firstly, the dominant points of the edges of the object (corner points) are determined while objects are defined. Objects with the help of the dominant points compared to a polygon, then the number of edges or vertices are determined. The purpose of the dominant point, the desired object is to represent, using fewer points. Thus, in practice, it is realized large memory, and trading volume. The problem is how to select these points. According to number of dominant point of any object, all combinations of boundary pixels is tested. Thus the exact solution is used in polygonal approach that gives the least error. If the object is small, and the required number of points is less, the exact solution will give the best results. However, if the required number of combinations is more, deterministic solution is impossible. Therefore, for solving the problem it is needed a stochastic search algorithm are needed. In this case, artificial bee colony (ABC) algorithm selected. ABC algorithm has been developed by modeling the bees look for food in bulk [2]. In this study, the advantages and shortcomings of the ABC method were examined by comparing ABC method with Genetic algorithm method

Keywords: *Dominant Point, Digital Curve, Polygonal Approximation, Artificial Bee Colony Algorithm*

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Generating Random Points from Arbitrary Distribution In Polygonal Areas

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Abstract

Random numbers generation in polygonal area is used in many applications area simulation. These application areas can be distribution of living life in a pond, level of pollution in a city, density of tree species in a forest, traffic flow in a region, density of flight in air space, diversity of wildlife in a region, crime rate in a city etc. [1].

Traditionally, acceptance-rejection method is used that in a polygonal area which it has known probability density function (f(x,y)) to generate random numbers [2]. For this, at first rectangular area boundaries which are surrounding polygon is found. By the help of these boundaries X and Y random values are created which they selected from uniform distribution. If created point is out of polygonal area, point is rejected. If selected point within polygonal area for adapting selected point to probability density function, a random value which is between zero and highest probability density value within polygonal area selected from uniform distribution (z direction). If this value is bigger than value of f(X,Y) is rejected, if not it is accepted. Thus, a random point is selected in polygonal area. This procedure can be repeated any numbers of random numbers are generated. Used method not only generate unnecessary random number but also it causes increased computational time for investigating whether the point in polygonal area.

The basis of proposed method is based on that calculated by dividing triangle pieces of the all area with corner points of polygonal area by combining together. By selecting a certain random point in polygonal area, triangulation size can be reduced and calculation sensitization can be increased. A plane which is in the probability density function value of corner points of each triangle, regarded as probability density function of the triangle. Under the probability density functions that they have all triangles volume be equal 1 is agreed as the probability basic axiom. Hence volume of triangular prism of each triangle formed is gave probability that the selection of the triangle. The probability density function defines in a unit triangle which is subtending the probability density function for each triangle. A random number generated within this defined triangle is moved by the principle of affine invariance into selected triangle area. In this manner any number of random numbers can be generated.

The method proposed in this study not only prevents unnecessary random number generation but also reduces computation time indeed. Specially, when want to generate a large number of random numbers it can be used as an effective method.

Keywords: *Random Number, Poligonal Area, Triangulate*

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Panoramic Image Mosaicing Using Multi-Object Artificial Bee Colony Optimization Algorithm

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Abstract

Sometimes distance, necessary to take wide- angle photography , may not be available. In this case, the need may occur combining in accordance photographs taken piece by piece. Nowadays, a lot of camera manufacturer tried to solve the problem by using wide-angle lens (fish eye) [1-2]. But in order to change perspective it is almost impossible to get a good image. On the other hand, on the basis of images taken by a number of different angle (especially video images) may be required to obtain a wide-angle image. In this case, in accordance with a multi-image combined panoramic images are obtained. However, adaptation research for the realization of suitable attachment can take a very long time.

In this study, for solving the problem, artificial bee colony algorithm [3] is changed based on adopted multi-object search. According to this method, the right side of each image is determined as the food region and the left side represents a bee hive. The bees in each hive move to food regions of other images, divided into groups that have equal number of bees. Each bee has its own search on food regions. After one of the bees which from the first hive reached the highest value of the objective function, tries to pull the other bees from other regions. As a result of a particular iteration bees of every hive are kept together in a certain region. Thus, it can be determined that which image is positioned in which order and which location.

Keywords: *Multi-Object Optimization, Artificial Bee Colony Algorithm,Panoramic Image Mosaicing*

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Some Properties of a Sturm-Liouville-Type Problem and The Green Function

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Abstract

In this study we have created Hilbert Space of The Sturm-Liouville Boundary Value Problem in $[0, \pi]$ interval, with boundary conditions which has λ complex eigenparameter. We have shown symmetric of appropriate operator to the problem. We have obtained asymptotic of solution functions and asymtotic of wronskian of the solution functions by using them. Moreover, we have examined Green function and asymtotic expansion of eigenvalues.

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Real Time 3D Palmprint Pose Estimation and Feature Extraction Using Multiple View Geometry Techniques

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Abstract

In this paper, it was aimed to develop a system that works in real time for to obtain palmprint pose (point of view) of a fully opened hand towards the camera. This system will be both a platform independent model (non-touchable) and arising from the hand movement rotations, translations and scaling independet model. For this purpose, pointed at the same direction two cameras (stereo) is used instead of single-camera vision systems system. Palmprint informations carried to 3D space using Mutliple View Geometry techniques from the obtained images. Thus, the problems are eliminated in previous studies as rotation, translation, scaling and platform dependecy.

Common points must be identified and mapped for capture of 3D palmprint on obtained images from two cameras. SURF algorithm based on Hessian matrix is determined common interest points on real-time snapshots of each cameras. The Levenberg-Marquardt optimization algorithm is used to minimize deviations from the characteristics of the cameras. Paired interest points of palmprint was considered to be approximately on a plane. Normal of 3D plane will give palmprint pose (point of view) according to the cameras. Finally, the palmrint image were transferred to the 2D surface with affine transformation. As a result, palmprint patterns have been obtained for strong 2D recognition palmprint systems.

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Chaos in cubic-quintic nonlinear oscillator

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Abstract

In this paper, we used Hamiltonian formulation and Lie transform to investigate a strongly nonlinear oscillator. Using Chirikovâ€■s overlap criterion we find the value of ε_{cr} at which the chaos loses its local character and becomes global. The results of Lie transformation analysis and Chirikovâ€■s criteria for the oscillator are compared with numerically generated Poincare Maps.

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The Numerical Solution of Boundary Value Problems by using Galerkin Method

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Abstract

In this study, we obtain approximate solutions of some boundary value problems by the Galerkin method. To demonstrate the effectiveness of the Galerkin method, we give some examples. Also, we compare the obtained solutions and their exact solutions by using Mathematica.

Keywords: Boundary value problems (BVPs), Galerkin method, Mathematica

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Semismooth Newton method for gradient constrained minimization problem

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Abstract

In this paper we treat a gradient constrained minimization problem which has applications in mechanics and superconductivity [1, 2, 5]:

$$\begin{aligned}
&\text{Find a solution } y \in K \text{ such that} \qquad (1)\\
&J(y) = \min_{v \in K} J(v),\\
&\text{where } J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 dx - \int_\Omega f v dx,\\
&K = \{v \in H_0^1(\Omega) | \quad |\nabla v(x)| \leq 1 \text{ a.e. in } \Omega\}.
\end{aligned}$$

Here $\Omega \subset \mathbf{R}^n$, $n \leq 3$, is bounded Lipschitz domain and $f \in L^2(\Omega)$ is given. A particular case of this problem is the elasto-plastic torsion problem.

In order to get the numerical approximation to the solution we have developed an algorithm in an infinite dimensional space framework using the concept of the generalized, so called, Newton differentiation [3,4,6]. At first we regularize the problem in order to approximate it with the unconstrained minimization problem and to make the pointwise maximum function Newton differentiable. Afterwards, using semismooth Newton method, we obtain continuation method in function space. For the numerical implementation the variational equations at Newton steps are discretized using finite elements method. We compare the numerical results in two-dimensional case obtained using C^1-conforming and non-conforming finite elements discretization.

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The Finite Element Method Solution of Variable Diffusion Coefficient Convection-Diffusion Equations

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Abstract

Mathematical modeling of many physical and engineering problems is defined with convection-diffusion equation. Therefore, there are many analytic and numeric studies about convection-diffusion equation in literature. The finite element method is the most preferred numerical method in these studies since it can be applied to many problems easily. But, most of the studies in literature are about constant coefficient case of the convection-diffusion equation. In this study, the finite element formulation of the variable coefficient case of the convection-diffusion equation is given in both one and two dimensional cases. Accuracy of the obtained formulations are tested on some problems in one and two dimensions.

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Multiple solutions for quasilinear equations depending on a parameter

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Abstract

The purpose of this talk is to use a very recent three critical points theorem due to Bonanno and Marano [1] to establish the existence of at least three solutions for quasilinear second order differential equations on a compact interval $[a, b] \subset R$ under appropriate hypotheses. We exhibit the existence of at least three (weak) solutions and, and the results are illustrated by examples.

Keywords- Dirichlet problem; Critical point; Three solutions; Multiplicity results.
AMS subject classification: 34B15; 47J10.

1 Main results

Consider the following quasilinear two-point boundary value problem

$$\begin{cases} -u'' = (\lambda f(x, u) + g(u))h(u') & \text{in } (a, b), \\ u(a) = u(b) = 0 \end{cases} \tag{1}$$

where $[a, b] \subset R$ is a compact interval, $f : [a, b] \times R \to R$ is an L^1-Carat$\acute{e}$odory function, $g : R \to R$ is a Lipschitz continuous function with $g(0) = 0$, i.e., there exists a constant $L \geq 0$ provided

$$|g(t_1) - g(t_2)| \leq L|t_1 - t_2|$$

for all $t_1, t_2 \in R$, $h : R \to]0, +\infty[$ is a bounded and continuous function with $m := \inf h > 0$ and λ is a positive parameter.

Employing Theorem 3.6 of [1], we establish the existence of at least three distinct (weak) solutions in $W_0^{1,2}([a, b])$ to the problem (1) for any fixed positive parameter λ belonging to an exact interval which will be observed in the main results.

We mean by a (weak) solution of problem (1), any $u \in W_0^{1,2}([a,b])$ such that

$$\int_a^b u'(x)v'(x)dx - \int_a^b [\lambda f(x,u(x)) + g(u(x))]h(u'(x))v(x)dx = 0$$

for every $v \in W_0^{1,2}([a,b])$. Denote $M := \sup h$ and suppose that the constant $L \geq 0$ satisfies $LM(b-a)^2 < 4$.

We introduce the functions $F : [a,b] \times R \to R$, $H : R \to R$ and $G : R \to R$ respectively, as follows

$$F(x,t) = \int_0^t f(x,\xi)d\xi \ \text{ for all } (x,t) \in [a,b] \times R,$$

$$H(t) = \int_0^t \int_0^\tau \frac{1}{h(\delta)} d\delta d\tau \ \text{ for all } t \in R$$

and

$$G(t) = -\int_0^t g(\xi)d\xi \ \text{ for all } t \in R.$$

We now formulate our main result.

Theorem 1. Assume that there exist a positive constant r and a function $w \in W_0^{1,2}([a,b])$ such that

(α_1) $\int_a^b [G(w(x)) + H(w'(x))]dx > r$,

(α_2) $\dfrac{\int_a^b \sup_{t \in [-\sqrt{\frac{2M(b-a)r}{4-LM(b-a)^2}}, \sqrt{\frac{2M(b-a)r}{4-LM(b-a)^2}}]} F(x,t)dx}{r} < \dfrac{\int_a^b F(x,w(x))dx}{\int_a^b [G(w(x))+H(w'(x))]dx}$,

(α_3) $\limsup_{|t| \to +\infty} \frac{F(x,t)}{t^2} < \frac{4-LM(b-a)^2}{2M(b-a)^2 r} \int_a^b \sup_{t \in [-\sqrt{\frac{2M(b-a)r}{4-LM(b-a)^2}}, \sqrt{\frac{2M(b-a)r}{4-LM(b-a)^2}}]} F(x,t)dx$ uniformly respect to $x \in [a,b]$.

Then, for each

$$\lambda \in \Lambda_1 := \left] \frac{\int_a^b [G(w(x)) + H(w'(x))]dx}{\int_a^b F(x,w(x))dx}, \frac{r}{\int_a^b \sup_{t \in [-\sqrt{\frac{2M(b-a)r}{4-LM(b-a)^2}}, \sqrt{\frac{2M(b-a)r}{4-LM(b-a)^2}}]} F(x,t)dx} \right[$$

the problem (1) admits at least three distinct weak solutions in $W_0^{1,2}([a,b])$.

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The Modified Bi-quintic B-spline base functions: An Application to Diffusion Equation

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Abstract

In this paper, the bi-quintic B-spline base functions are modified on a general 2-dimensional problem and then they are applied to two-dimensional Diffusion problem in order to obtain its numerical solutions. The computed results are compared with the results given in the literature. The error norms L_2 and L_∞ are computed and found to be marginally accurate and efficient.

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Numerical Solutions of the Modified Burgers' Equation by Cubic B-spline Collocation Method

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Abstract

In this paper, a numerical solution of the modified Burgers' equation is obtained by a cubic B-spline collocation method. In the solution process, a linearization technique has been applied to deal with the non-linear term appearing in the equation. The computed results are compared with the results given in the literature. The error norms L_2 and L_∞ are also computed and found to be sufficiently small.

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The Modified Kudryashov Method for Solving Some Evolution Equations

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Abstract

The study of numerical methods for solving partial differential equations and the travelling wave solutions of these equations have significant roles in physical science over the last decades from both theoretical and the practical points of view. Mathematical physics consist of many mathematical models which described by the nonlinear partial differential equations. The investigation of the travelling wave solutions of nonlinear evolution equations appears in various scientific fields, such as plasma physics, fluid mechanics, hydrodynamic, optical fibers, chemical physics. Many powerful and effective methods are used for investigating the explicit travelling wave solutions.

In this paper, we have applied the modified Kudryashov method for solving some nonlinear evolution equations by the help of commutative algebra. This method is applicable for the other nonlinear partial differential equations.

We consider the general nonlinear partial differential equation for a function u of two variables, space x and time t:

$$P(u,u_t,u_x,u_{xx},u_{tt},u_{xt},...)=0 \tag{1}$$

It is useful to summarize the steps of modified Kudryashov method as follows[5]:

Step 1. We investigate the travelling wave solutions of Eq.(1) of the form:

$$u(x,t)=u(\xi), \qquad \xi = kx+wt, \tag{2}$$

where k and w are arbitrary constants. Then Eq.(1) reduces to a nonlinear ordinary differential equation of the form:

$$G(u,u_\xi,u_{\xi\xi},u_{\xi\xi\xi},...)=0 \tag{3}$$

Step 2. We suppose that the exact solutions of Eq.(3) can be obtained in the following form:

$$u(\xi) = y(\xi) = \sum_{i=0}^{N} a_i Q^i, \quad (4)$$

where $Q = \dfrac{1}{1+e^{\xi}}$ and the function Q is the solution of equation

$$Q_{\xi} = Q^2 - Q \quad (5)$$

Step 3. According to the method, we assume that the solution of Eq.(3) can be expressed in the form

$$u(\xi) = a_N Q^N + \ldots \quad (6)$$

Calculation of value N in formula (6) that is the pole order for the general solution of Eq. (3). In order to determine the value of N we balance the highest order nonlinear terms in Eq. (3) analogously as in the classical Kudryashov method. Supposing $u^l(\xi)u^{(s)}(\xi)$ and $(u^p(\xi))^r$ are the highest order nonlinear terms of Eq. (3) and balancing the highest order nonlinear terms we have:

$$N = \frac{s - rp}{r - l - 1}, \quad (7)$$

Step 4. Substituting Eq.(4) into Eq.(3) and equating the coefficients of Q^i to zero, we get a system of algebraic equations. By solving this system, we obtain the exact solutions of Eq.(1).

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Study of an inverse problem that models the detection of corrosion in metalic plate whose lower part is embedded

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Abstract
In this work, we will study an inverse problem to determine corrosion in an inaccessible location of a metalic plate. Our study area is inside the plate metalic plate whose lower part is embedded therefore inacssecible. We will perform measurements on the upper part of the plate, which is not in contact with the ground. For this, we will send an electric field on this part and take measurements. This problem is modeled by a Laplace problem with mixed presence of an unknown term in the boundary conditions this term is an unknown function which can take several forms. It is this function that we will detect the presence or absence of corrosion inside the tube and we will then follow our steps to the top edge of the field information on the evolution of this corrosion. We will first formulate our problem which is an inverse problem and we will make a theoretical study and we will that this problem has a unique solution also this solution is stable. After, we will solve this problem by constructing an iterative algorithm which gives problems that will cross a series of impedance functions which determines the rate of corrosion. Finally we study the convergence and we will then make a numerical application

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Commuting nilpotent operators with maximal rank

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Abstract

Let X, $\hat{X}$ be commuting nilpotent matrices over a field k with nilpotency p^t. We show that if $X - \hat{X}$ is a certain linear combination of products of commuting nilpotent matrices, then X is of maximal rank if and only if $\hat{X}$ is of maximal rank. In the case, k is an algebraically closed field of positive characteristic p there is an interpretation about module over group algebras.

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Finding Global minima with a new class of filled function

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Abstract

Global Optimization problems arise in many fields of science and technology [2-4]. Filled function method is a type of efficient methods to obtain the global solution of a multivariable function. The key idea of the filled function method is to leave from a current local minimizer x^* to a lower minimizer x^* of the original objective function $f(x)$ with the auxiliary function $P(x)$ constructed at the local minimizer. This method was introduced in Ge's paper [1] for continuous global optimization problem, the first filled has the form

$$p(x, r, \rho) = \frac{1}{r + f(x)} \exp\left(-\frac{\|x - x_k^*\|^2}{\rho^2}\right)$$

where r and ρ are two adjustable parametres.

This paper gives a new definition of the filled function. It shows that the filled function given in some paper are the special forms of this filled functions.

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Weak Convergence Theorem For A Semi-Markovian Random Walk With Delay And Pareto Distributed Interference Of Chance

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Abstract

In this study, a semi-Markovian random walk with delay and a discrete interference of chance $(X(t))$ is constructed. The weak convergence theorem is proved for the ergodic distribution of the process $X(t)$ and the limit form of the ergodic distribution is found, when the random variables $\{\zeta_n\}, n \geq 0$ have Pareto distribution with parameters (α, λ) where the random variables ζ_n describe the discrete interference of chance.

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PARAMETER DEPENDENT NOVIER-STOKES LIKE PROBLEMS

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Abstract

In this talk, the following nonstationary Novier-Stokes like equation with variable coefficients

$$\frac{\partial u}{\partial t}-A_{\varepsilon}\left(x\right)u+\left(u.\nabla\right)u+\nabla\varphi=f\left(x,t\right),\ \operatorname{div}u=0,\ x\in G,\ t\in\left(0,T\right),$$

$$L_{1\varepsilon}u=\sum_{i=0}^{\nu}\varepsilon^{\sigma_i}\alpha_i\frac{\partial^i u}{\partial x_n^i}\left(x',0,t\right)=0,\ \nu\in\left\{0,1\right\},$$

$$u\left(x,0\right)=a\left(x\right),\ x\in R_+^n,\ t\in\left(0,T\right),$$

is considered, where

$$R_+^n=\left\{x\in R^n,\ x_n>0,\ x=\left(x',x_n\right),\ x'=\left(x_1,x_2,...,x_{n-1}\right)\right\},$$

$$A_{\varepsilon}\left(x\right)u=\varepsilon\sum_{k=1}^{n}a_k\left(x\right)\frac{\partial^2 u}{\partial x_k^2},\ \sigma_i=\frac{1}{2}\left(i+\frac{1}{q}\right),\ q\in\left(1,\infty\right),$$

ε is a small positive parameter, α_i are complex numbers, a_k are continious functions on R_n^n,

$$u=u_{\varepsilon}\left(x\right)=\left(u_{1\varepsilon}\left(x,t\right),u_{2\varepsilon}\left(x,t\right),...,u_{n\varepsilon}\left(x,t\right)\right)$$

are represent the unknown velocity, $f=\left(f_1\left(x,t\right),f_2\left(x,t\right),...,f_n\left(x,t\right)\right)$ represents a given external force and a denotes the initial velocity.

The existence, uniqueness and L^p estimates of solution the above problem is derived.

A New Spline Approximation for the Solution of One-space Dimensional Second Order Non-linear Wave Equations With Variable Coefficients

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Abstract: In this paper, we propose a new three-level implicit nine point compact finite difference formulation of order two in time and four in space directions, based on non-polynomial spline in compression for the solution of one-space dimensional second order non-linear hyperbolic partial differential equations with variable coefficients and significant first order space derivative term. We describe the Mathematical formulation procedure in details and also discussed the stability. Numerical results are provided to justify the usefulness of the proposed method.

Keywords: Non-polynomial spline in compression; Non-linear Wave equation; Maximum absolute errors

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An error correction method for solving stiff initial value problems based on a cubic C^1-spline collocation method

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Abstract

For solving nonlinear stiff initial value problems, we develop an improved error correction method (IECM) which originates from the error corrected Euler methods (ECEM) recently developed by the authors (see [17, 18]) and reduces the computational cost and further enhances the stability for the ECEM. We use the stabilized cubic C^1-spline collocation method instead of the Chebyshev collocation method used in ECEM for solving the asymptotic linear ODE for the difference between the Euler polygon and the true solution. It is proved that IECM is A-stable, a semi-implicit one-step method, and of order 4 with only one evaluation of the Jacobian at each integration step. Also, we use the iteration process of the Lobatto IIIA method developed by [13] for solving the induced matrix system. It is shown that this iteration process does not require such the nonlinear function evaluation as the implicit method does and hence it reduces the numerical computational cost efficiently. Numerical evidence is provided to support the theoretical results with several stiff problems.

Keywords: Euler polygon, Cubic C^1-spline collocation method, Lobatto IIIA method, Error correction method, Stiff initial value problem

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[1]This work was supported by basic science research program through the National Research Foundation of Korea(NRF) funded by the ministry of education, science and technology (grant number 2011-0029013).

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On Numerical Solution of Multipoint NBVP for Hyperbolic-Parabolic Equations with Neumann Condition

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Abstract

Certain problems of modern physics and technology can be effectively described in terms of nonlocal problem for partial differential equations. These nonlocal conditions arise mainly when the data on the boundary cannot be measured directly. Methods of solutions of nonlocal boundary value problems for partial differential equations and partial differential equations of mixed type have been studied extensively by many researchers in [1-5].

In this paper, numerical solutions of difference schemes of multipoint nonlocal boundary value problem for multidimensional hyperbolic-parabolic equation with Neumann condition are considered. The first and second orders of accuracy difference schemes are established. The theoretical statements for the solution of these difference schemes are supported by results of numerical experiments.

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Classification of exact solutions for the Pochhammer-Chree equations

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Abstract

In this study, exact solutions to the Pochhammer-Chree equations are obtained by using complete discrimination system. These solutions can be reduced to soliton solution, rational and elliptic function solutions. Also, we propose a more general method for the generalized nonlinear partial differential equations.

Keywords: Trial equation method, Soliton solutions, Elliptic function solutions.

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On the Density of Regular Functions in Variable Exponent Sobolev Spaces

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Abstract

The talk will deal with when every function in a variable exponent Sobolev space can be approximated by a more regular function, such as a smooth or Lipschitz continuous function. Many researchers have made contributions , but still remain substantial gaps in our understanding of this intricate question. A discussion on methods also will take place. I will also give some my results for the density in variable exponent Sobolev spaces .

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Numerical Solution of a Hyperbolic-Schrödinger Equation with Nonlocal Boundary Conditions

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Abstract

A numerical method is proposed for solving hyperbolic-Schrödinger partial differential equations with nonlocal boundary condition. The first and second orders of accuracy difference schemes are presented. A procedure of modified Gauss elimination method is used for solving these difference schemes in the case of a one-dimensional hyperbolic-Schrödinger partial differential equations. The method is illustrated by numerical examples.

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New generalized hyperbolic functions to find exact solution of the nonlinear partial differential equation

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Abstract

In this article, we first time define new functions (called generalized hyperbolic functions) and devise new kinds of transformation (called generalized hyperbolic function transformation) to construct new exact solutions of nonlinear partial differential equations. Based on the generalized hyperbolic function transformation of the generalized KdV equation. We obtain abundant families of new exact solutions of the equation and analyze the properties of this by taking different parameter values of the generalized hyperbolic functions. As a result, we find that these parameter values and the region size of the independent variables affect some solution structure. These solutions may be useful to explain some physical phenomena.

1 Introduction

To construct exact solutions to nonlinear partial differential equations, some important methods have been defined such as Hirota method, tanh-coth method, the exponential function method, (G'/G)-expansion method, the trial equation method, and so on [1-15]. There are a lot of nonlinear evolution equations that are integrated using the various mathematical methods. Soliton solutions, compactons, singular solitons and other solutions have been found by using these approaches. These types of solutions are very important and appear in various areas of applied mathematics. In Section 2, we give the definition and properties of generalized hyperbolic functions. In Section 3, as applications, we obtain exact solution of the generalized KdV equation

$$(u^l)_t + \alpha u(u^n)_x + \beta[u(u^n)_{xx}]_x + \gamma u(u^n)_{xxx} = 0. \tag{1}$$

2 The definition and properties of the symmetrical hyperbolic Fibonacci and Lucas functions

In this section, we will define new functions which named the symmetrical hyperbolic Fibonacci and Lucas functions for constructing new exact solutions of NPDEs, and then study the properties of these functions.

Definition 2.1 Suppose that ξ is an independent variable, p, q and k are all constants. The generalized hyperbolic sine function is

$$\sinh_a(\xi) = \frac{pa^{k\xi} - qa^{-k\xi}}{2}, \tag{2}$$

generalized hyperbolic cosine function is

$$\cosh_a(\xi) = \frac{pa^{k\xi} + qa^{-k\xi}}{2}, \tag{3}$$

generalized hyperbolic tangent function is

$$\tanh_a(\xi) = \frac{pa^{k\xi} - qa^{-k\xi}}{pa^{k\xi} + qa^{-k\xi}}, \tag{4}$$

generalized hyperbolic cotangent function is

$$\coth_a(\xi) = \frac{pa^{k\xi} + qa^{-k\xi}}{pa^{k\xi} - qa^{-k\xi}}, \tag{5}$$

generalized hyperbolic secant function is

$$sech_a(\xi) = \frac{2}{pa^{k\xi} + qa^{-k\xi}}, \tag{6}$$

generalized hyperbolic cosecant function is

$$cosech_a(\xi) = \frac{2}{pa^{k\xi} - qa^{-k\xi}}, \tag{7}$$

the above six kinds of functions are said generalized new hyperbolic functions. Thus we can prove the following theory of generalized hyperbolic functions on the basis of Definition 2.1.

Theorem 2.1. The generalized hyperbolic functions satisfy the following relations:

$$\cosh_a^2(\xi) - \sinh_a^2(\xi) = pq, \tag{8}$$

$$1 - \tanh_a^2(\xi) = pq.sech_a^2(\xi), \tag{9}$$

$$1 - \coth_a^2(\xi) = -pq.cosech_a^2(\xi), \tag{10}$$

$$sech_a(\xi) = \frac{1}{\cosh_a(\xi)}, \tag{11}$$

$$cosech_a(\xi) = \frac{1}{\sinh_a(\xi)}, \tag{12}$$

$$\tanh_a(\xi) = \frac{\sinh_a(\xi)}{\cosh_a(\xi)}, \tag{13}$$

$$\coth_a(\xi) = \frac{\cosh_a(\xi)}{\sinh_a(\xi)}. \tag{14}$$

The following just part of them are proved here for simplification.

Theorem 2.2. The derivative formulae of generalized hyperbolic functions as following

$$\frac{d(\sinh_a(\xi))}{d\xi} = k \ln a \ \cosh_a(\xi), \tag{15}$$

$$\frac{d(\cosh_a(\xi))}{d\xi} = k \ln a \ \sinh_a(\xi), \tag{16}$$

$$\frac{d(\tanh_a(\xi))}{d\xi} = kpq \ln a \ sech_a^2(\xi), \tag{17}$$

$$\frac{d(\coth_a(\xi))}{d\xi} = -kpq \ln a \ cosech_a^2(\xi), \tag{18}$$

$$\frac{d(sech_a(\xi))}{d\xi} = -k \ln a \ sech_a(\xi) \tanh_a(\xi), \tag{19}$$

$$\frac{d(cosech_a(\xi))}{d\xi} = -k \ln a \ cosech_a(\xi) \coth_a(\xi). \tag{20}$$

Proof of (17): According to (15) and (16), we can get

$$\frac{d(\tanh_a(\xi))}{d\xi} = \left(\frac{\sinh_a(\xi)}{\cosh_a(\xi)}\right)' = \frac{(\sinh_a(\xi))' \cosh_a(\xi) - (\cosh_a(\xi))' \sinh_a(\xi)}{\cosh_a^2(\xi)}$$

$$= \frac{k \ln a \ \cosh_a^2(\xi) - k \ln a \ \sinh_a^2(\xi)}{\cosh_a^2(\xi)} = kpqsech_(^2\xi), \tag{21}$$

Similarly, we can prove other differential coefficient formulae in Theorem 2.2.

Remark 2.1. We see that when $p = 1$, $q = 1$, $k = 1$ and $a = e$ in (2)-(7), new generalized hyperbolic function $\sinh_a(\xi)$, $\cosh_a(\xi)$, $\tanh_a(\xi)$, $\coth_a(\xi)$, $sech_a(\xi)$ and $cosech_a(\xi)$, degenerate as hyperbolic function $\sinh(\xi)$, $\cosh(\xi)$, $\tanh(\xi)$, $\coth(\xi)$, $sech(\xi)$ and $cosech(\xi)$, respectively. In addition, when $p = 0$ or $q = 0$ in (2)-(7), $\sinh_a(\xi)$, $\cosh_a(\xi)$, $\tanh_a(\xi)$, $\coth_a(\xi)$, $sech_a(\xi)$ and $cosech_a(\xi)$, degenerate as exponential function $\frac{1}{2}pa^{k(\xi)}$, $\pm\frac{1}{2}qa^{-k(\xi)}$, $2pa^{-k(\xi)}$, $\pm 2qa^{k(\xi)}$ and ± 1, respectively.

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EQUIVALENCE OF AFFINE CURVES

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ABSTRACT

The definitions of affine curve, affine arclength and affine types of an affine curve are given in R^2. Affine invariant parametrization of an affine curve which is invariant under the affine group is introduced. The complete system of affine differential invariants for affine plane curves is obtained and we show that these invariants are independent. The conditions of equivalence of two affine curves is obtained in terms of affine differential invariants in R^2.

Keywords: Affine geometry, affine curve, affine differential invariants, affine equivalence.
MSC: 53A15, 53A55.

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Modified trial equation method for nonlinear differential equations

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Abstract

In this research, we defined a new approach with respect to the trial equation method. This method is applied for constructing the soliton solutions, rational function solutions and elliptic function solutions. Also, we conclude that the modified trial equation method can be extended to solve other physical problems in nonlinear science.

Keywords: Modified trial equation method, Soliton solutions, Elliptic function solutions.

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Finite Difference Method for the Integral-Differential Equation of the Hyperbolic Type

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Abstract.

Hyperbolic partial differential equations are used in many branches of physics, engineering and several areas of science, e.g. electromagnetic, electrodynamic, hydrodynamics, elasticity, fluid flow and wave propagation [1, 2, 3]. There is a great deal of work for solving these type of problems numerically and their stability in various functional spaces has received a great deal of importance. However, most of these works are studied in one-dimensional space (see [4, 5, 6, 7]). There are some studies about the numerical solution of two-dimensional hyperbolic equations with the collocation method or rational differential quadrature method [8, 9].

Let Ω be the unit open cube in the n-dimensional Euclidean space $\mathbb{R}^n$ $(0 < x_k < 1,\ 1 \le k \le n)$ with the boundary S, $\overline{\Omega} = \Omega \cup S$. In $[-1,1]\times\Omega$ the mixed problem for the multidimensional integral-differential equation of the hyperbolic type

$$\begin{cases} v_{tt} - \sum\limits_{r=1}^{n} (a_r(x)V_{x_r})_{x_r} = \int\limits_{-t}^{t} \sum\limits_{r=1}^{n} (b_r(p,x)v_{x_r})_{x_r}\, dp + f(t,x), & -1 \le t \le 1,\ x=(x_1,\ldots,x_n)\in\Omega, \\ v(t,x) = 0, \quad x\in S,\ -1\le t\le 1, & \\ v(0,x)=\varphi(x),\ v_t(0,x)=\psi(x), \quad x\in\overline{\Omega} & \end{cases} \tag{1}$$

is considered. In [10] it was proved that the problem (1) has a unique smooth solution $v(t,x)$ for the smooth functions $a_r(x) \ge \delta > 0$, $r = 1,\ldots,n$, $\varphi(x)$, $\psi(x)$, $x \in \overline{\Omega}$ and $f(t,x)$, $b(t,x)$, $t \in (-1,1)$, $x \in \Omega$. Moreover, the first order of accuracy difference scheme was investigated.

In the present paper the second order of accuracy difference scheme approximately solving the problem (1) is studied. The stability estimates for the solution of this difference scheme are established. Theoretical results are supported by numerical examples.

Keywords: Finite Difference Method; Integral-Differential Equation of the Hyperbolic Type
PACS: 02.60.Lj, 02.60.Nm, 02.70.Bf, 87.10.Ed

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Normal Extensions of a Singular Differential Operator For First Order

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Abstract

In this work, in terms of boundary values all normal extensions of the minimal operator generated by a linear singular differential-operator expression for first order with operator coefficients in Hilbert space of vector-functions in a right half-infinite interval are described. Later on, a point spectrum of such extensions has been investigated.

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Reproducing Kernel Hilbert Space Method for Solving the Pollution Problem of Lakes

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Abstract

Pollution is a major threat for our environment. Monitoring pollution is the first step to save environment and has become possible with use of differential equations. This study includes the problem of pollution of three lakes connected with pipes or channels [4]. Consider the following mathematical model describing the pollution of a system of lakes [1-3] :

$$\begin{cases} \frac{dx_1}{dt} = \frac{F_{13}}{V_3}x_3(t) + p(t) - \frac{F_{31}}{V_1}x_1(t) - \frac{F_{21}}{V_1}x_1(t) \\ \frac{dx_2}{dt} = \frac{F_{21}}{V_1}x_1(t) - \frac{F_{32}}{V_2}x_2(t) \\ \frac{dx_3}{dt} = \frac{F_{31}}{V_1}x_1(t) + \frac{F_{32}}{V_2}x_2(t) - \frac{F_{13}}{V_3}x_3(t) \end{cases} \tag{1}$$

The approximate solutions are obtained with Reproducing Kernel Hilbert Space Method [5-6] for three different models: impulse, step and sinusoidal. The absolute errors are calculated by comparing the numerical results to the analytic results. The errors are seen to be acceptable. All of the numerical computations have been calculated on a computer programme with MATHEMATICA .

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The q analogue of the limit case of bernstein type operators

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Abstract In the present paper we introduce a q-analogue of the Bernstein-typeoperators which is defined in cite{4}. We estimate moments,

establish direct theorems and rate of convergence in terms of themodulus of continuity.

In 1997 Philips [1] proposed the following q-analogue of the well-known Bernstein polynomials,which for each positive integer n and $f \in C[0,1]$,are defined as,

$$B_{n,q}(f;x) = \sum_{k=0}^{n} f\left(\frac{[k]}{[n]}\right) p_{nk}(q;x).$$

After Philips several researchers have studied convergence properties of q-Bernstein polynomials $B_{n,q}(f;x)$. We can refer to readers these important searchs in $[12, 13, 14, 15]$.

P.E. Parvanov , B. D. Popov in 1994 mention Bernstein type operators and examined direct theorems and Jackson type inequality and some approximation properties. This motives us to examine and introduce q analogue of Bernstein type operators.

the class of q-Bernstein operators discussed in this paper are given for natural n by

$$\begin{aligned} U_{n,q}(f;x) &= [n-1] \sum_{k=1}^{n} q^{1-k} p_{nk}(q;x) \int_0^1 f(t)\, p_{n-2,k-1}(q;qt)\, d_q t + f(0)\, p_{n,0}(q;x) \\ &= \sum_{k=1}^{n} b_{nk}(q;x)\, p_{n,k}(q;x). \end{aligned} \tag{1}$$

where $p_{n,k}(q;x) = \left[{n \atop k}\right] x^k (1-x)_q^{n-k}$ and the quantity $b_{nk}(q;x) = q^{1-k} \int_0^1 [n-1]\, f(t)\, p_{n-2,k-1}(q;qt)\, d_q t$ for $1 \le k \le n$ in the operators $U_{n,q}(f;x)$ takes place of $f\left[{k \atop n}\right]$ in $B_{n,q}(f;x)$ the Bernstein polynomials and $b_{nk}(q;x)$ satisfies $b_{n,0}(f) = f(0)$ and $b_{n,n}(f) = f(1)$.

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