

On the Classifications of C^* -Algebras Using Unitary Groups

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Abstract

In 1955, Dye proved that the discrete unitary group in a factor determines the algebraic type of the factor. Using Dye's approach, we prove similar results to a larger class of amenable unital C^* -algebras including simple unital AH-algebras (of SDG) with real rank zero. If φ is an isomorphism between the unitary groups of two unital C^* -algebras, it induces a bijective map θ_φ between the sets of projections of the algebras. For some UHF-algebras, we construct an automorphism φ of their unitary group, such that θ_φ does not preserve the orthogonality of projections. For a large class of unital C^* -algebras, we show that θ_φ is always an orthoisomorphism. This class includes in particular the Cuntz algebras \mathcal{O}_n , $2 \leq n \leq \infty$, and the simple unital AF-algebras having 2-divisible K_0 -group. If φ is a continuous automorphism of the unitary group of a UHF-algebra A , we show that φ is implemented by a linear or a conjugate linear *-automorphism of A .

References

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