Semismooth Newton method for gradient constrained minimization problem

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Abstract

In this paper we treat a gradient constrained minimization problem which has applications in mechanics and superconductivity [1, 2, 5]:

$$
\begin{aligned}
&\text{Find a solution } y \in K \text{ such that} \qquad (1)\\
&J(y) = \min_{v \in K} J(v),\\
&\text{where } J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2 dx - \int_\Omega f v dx,\\
&K = \{v \in H_0^1(\Omega) | \quad |\nabla v(x)| \leq 1 \text{ a.e. in } \Omega\}.
\end{aligned}
$$

Here $\Omega \subset \mathbf{R}^n$, $n \leq 3$, is bounded Lipschitz domain and $f \in L^2(\Omega)$ is given. A particular case of this problem is the elasto-plastic torsion problem.

In order to get the numerical approximation to the solution we have developed an algorithm in an infinite dimensional space framework using the concept of the generalized, so called, Newton differentiation [3,4,6]. At first we regularize the problem in order to approximate it with the unconstrained minimization problem and to make the pointwise maximum function Newton differentiable. Afterwards, using semismooth Newton method, we obtain continuation method in function space. For the numerical implementation the variational equations at Newton steps are discretized using finite elements method. We compare the numerical results in two-dimensional case obtained using C^1-conforming and non-conforming finite elements discretization.

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