# A Note on Some Elementary Geometric Inequalities

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**Abstract**

In present paper, solutions of some elementary geometric inequalities are obtained. Firstly, we get a more useful inequality by specifying the largest lower and smallest upper bounds, to be able to end the inaccuracy of the following inequality. Let a, b, c are lengths of sides of a triangle, and if distances of a taking point in the inner region of a triangle to the vertices are x, y, z, then following inequality satisfies

$$\frac{1}{2}\left(a+b+c\right) < x+y+z < a+b+c.$$

Nevertheless it is true and useful, but it has not accurate boundaries. Because neither $\frac{1}{2}\left(a+b+c\right)$ is the largest lower bound, nor $a+b+c$ is the smallest upper bound of this sum. But in university preparation course books and textbooks, which describe this inequality, they resolved by accepted these greatest lower bound and least upper bound, so that incorrect results were obtained. In this work, we solved this inequality with more useful bounds. Namely,

1. *If distances of a taking point in the inner region of a triangle to the vertices are x, y, z which have lengths of sides a, b, c, and the area A, and also measures of all internal angles are smaller than 120 degrees, then following inequality satisfies*

$$\sqrt{\frac{1}{2}\left(a^2+b^2+c^2+4\sqrt{3}A\right)} \leq x+y+z < \max\left\{a+b, a+c, b+c\right\}.$$

2. *If distances of a taking point in the inner region of a triangle to the vertices are x, y, z which have lengths of sides a, b, c, and the area A, and one of the measure of internal angle is greater than or equal to 120 degrees, then following inequality satisfies*

$$\min\left\{a+b, a+c, b+c\right\} < x+y+z < \max\left\{a+b, a+c, b+c\right\}.$$

We have stated and proved some theorems and lemmas that have done before (see in [1-5]). One of the our theorems is famous one, namely the Toricelli-Fermat point that was solved in many ways.
In [6] and [7], we observed that Mustafa Yağcı have studied such as this work nicely. But, our proofs are completely different and some parts are more simple and clearer. In [7], he gave a problem. He claimed the problem he has given can not be solved using only geometry, but calculus is also used to solve the problem. We decided to prove this problem given in [7]. After showing the existence and uniqueness of the triangle $\triangle XYZ$ defined in the problem, we tried to solve the problem by using only geometrical way and we succeeded. The most interesting part of our article is second part, proving this following problem given below:

**Problem.** *For a given point P, on changing plane of the X, Y and Z let be* $|PX| = a$, $|PY| = b$, and $|PZ| = c$. *Then the circumference of the triangle $\triangle XYZ$ has the largest value, when P is at the inner center of the triangle. Secondly, what can be the minimum value of the sum of a, b and c?*

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