A Perturbation Solution Procedure for a Boundary Layer Problem

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Abstract

A perturbation algorithm using a new transformation is introduced for boundary-value problem with small parameter multiplying the derivative terms. To account for the linear and the nonlinear dependence of the function, we exhibit the function f for the system. We introduce the transformation $T_e = f(x,\gamma;\varepsilon)\cdot x$, where f depends on x,γ and ε . Results of Multiple Scales method, method of matched asymptotic expansions and our method are contrasted.

We consider the following boundary-value problem

$$\begin{aligned} &\varepsilon y'' + y' + y^2 = 0 \\ &y(0) = 0 \qquad y(1) = \frac{1}{2} \end{aligned} \tag{1}$$

Where ε is a small dimesionless positive number. It is assumed that the equation and boundary conditions have been made dimensionless.

In Direct Perturbation Method, secular terms appear of higher orders of the expansion invalidating the solution. In order the avoid this problem a new transformation has been proposed in our study.

The new transformation is defined as,

$$T_e = f(x,\gamma;\varepsilon)\cdot x \tag{2}$$

Using the chain rule, we transform the derivate accordingly

$$\begin{aligned} &\frac{dy}{dx} = \frac{dy}{dT_e}\frac{dT_e}{dx} = \frac{dy}{dT_e}\left(\frac{df}{dx}x + f\right) = y'(f_x x + f) \\ &\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dT_e}\left(\frac{df}{dx}x + f\right)\right) = \left[\frac{d}{dT_e}\left(\frac{dy}{dT_e}\right)\right]\left(\frac{df}{dx}x + f\right)\frac{dT_e}{dx} + \frac{dy}{dT_e}\left[\frac{d}{dx}\left(\frac{df}{dx}x + f\right)\right] \\ &= \frac{d^2y}{dT_e^{\,2}}\left(\frac{df}{dx}x + f\right)^2 + \frac{dy}{dT_e}\left(\frac{d^2f}{dx^2}x + 2\frac{df}{dx}\right) = y''(f_x x + f)^2 + y'(f_{xx}x + 2f_x) \end{aligned} \tag{3}$$

So, we have obtained a more effective parameter expression T_e without losing the original parameter x,γ and ε . Thus, speeding up and slowing down control of the time parameter will be available as in Method of Multiple Scales.

In Equation (3) first order derivatives according to new variable T_e appear in the second order derivative expressions according to original time variable x. So, we are able to have information about parameters of nonlinear differential equation and to interpret the results.

By this new transformation we have the advantages of both method of matched asymptotic expansions and Method of Multi Scales [1-6].

Using perturbation algorithm with new transformation, a more effective time expression without losing the original time parameter t have been obtained. Information about parameters of nonlinear differential equation and interpretation of the results has been achieved. Applying this transformation on boundary value problem the results obtained are compared with the results of the studies conducted to time.

References

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