The normal inverse Gaussian distribution: exposition and applications to modeling asset, index and foreign exchange closing prices

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Abstract

We expose the unique properties of the normal inverse Gaussian distribution (NIG) useful for modeling asset, index and foreign exchange closing prices. We further demonstrate that traditional beliefs in asset, index, and foreign exchange closing prices not being independently identically distributed random variables are fundamentally flawed. Best models are selected using a novel model selection strategy proposed by Käärik and Umbleja (2011). Our results show that closing prices of Baltika and Ekpress Grupp (companies trading on Tallinn stock exchange), FTSE100, GSPC and STI (major world indexes), CHF/JPY, USD/EUR, EUR/GBP, SAR/CHF, QAR/CHF and EGP/CHF (Foreign Exchange rates) can be modeled by NIG distribution. This means their underlying stochastic properties can fully be captured by NIG; very useful for predicting price movements, pricing models, underwriting and trading derivatives etc[1].

Keywords: NIG distribution, modeling, index, asset, foreign exchange, goodness of fits tests

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1 Introduction

Researchers from across the spectrum continue to grapple with the problem of describing the stochastic properties of stock market prices. Some have employed data mining techiques with little success, as evidenced by [9] who elaborates ten such techniques, talk less neural networks and particle optimization algorithms elaborated and employed in [2]. Some have defended theories on stock market prices not following random walk approximation [5,7]. Likewise, others have come up with models that advocate stock market prices may follow random walk approximations [3, 5], but none of which makes use of already existing theoretical probability distributions. Most try to define a model and test certain assumptions [3, 5].

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As well, some have been able to fit log return financial data with NIG distribution and have declared it a better fit than normal or Gaussian based models[8, 12], although very few statistical test were carried out to make such conclusions. Our approach uses a proposed model selection strategy which incorporates statistical tests aimed at choosing best models; the Käärik and Umbleja (2011) proposed model selection technique [6].

The normal inverse Gaussian distribution has been studied extensively in [1, 4, 10]. In this work, we present just the unique qualities that mimic closing prices and makes this theoretical probability distribution stand out in this context. We do this through the generalized inverse Gaussian distribution; a member of the class of generalized hyperbolic distributions introduced in 1977 by Ole E. Barndorff-Nielssen[1]. It should be pointed out that this distribution is an approximation to a random walk, thus modelling prices with it constitute a random walk approximation as will be demonstrated in this work.

Section two introduces a general Lévy process of which the NIG distribution is a subclass. This is to demonstrate that it has properties similar to closing prices. Then we outline NIG unique properties; after presenting it as a special case of generalized hyperbolic distribution (GHYP). Model selection strategy and analysis is the subject of section 3.

2 NIG distribution and its general characteristics

2.1 General Lévy process

A Lévy process is a continuous time stcochastic process $X = \{X_t : t > 0\}$ defined on the probability space (Ω, F, P) with the floowing basic characteristics:

1. $P(X_0) = 1$ i.e. the process starts at zero;
2. $\forall s, t \geq 0, X_{s+t} - X_t$ is distributed as X_s i.e. stationary increments;
3. $\forall s, t \geq 0, X_{s+t} - X_t$ is independent of X_u, $s \leq t \leq u$, i.e. independent increments;
4. $t \to X_t$ is a.s. right continuous with left limits.

These are all characteristics of closing prices as well as the NIG distribution [10].

2.2 NIG presentation through GHYP

A random variable Z has a GHYP distribution with parameters $(\lambda, \alpha, \beta, \delta, \mu)$ if the conditional distribution is equal to

$$Z|Y = y \sim N(\mu + \beta y, y) \tag{1}$$

where

$$Y \sim \frac{1}{2}\{\frac{\sqrt{\alpha^2 - \beta^2}}{\delta}\}^{\lambda/2} K_\lambda^{-1}(\sqrt{\delta\sqrt{\alpha^2 - \beta^2}}) y^{\lambda-1} exp\{-\frac{1}{2}((\alpha^2 - \beta^2)y + \delta^2 y^{-1})\}, y > 0, \tag{2}$$

$$K_\lambda(y) = \frac{1}{2}\int_0^\infty u^{\lambda-1} e^{-\frac{1}{2}y(u+u^{-1})} du, y > 0 \tag{3}$$

and $N(\mu+\beta y, y)$ is the normal distribution[2] with mean $\mu+\beta y$ and variance y. Thus Z has a probability density function [1, 10]

$$f(z;\lambda,\alpha,\beta,\delta,\mu) = a_\lambda(\alpha,\beta,\delta)\{\sqrt{(\delta^2+(z-\mu)^2)}\}^{\lambda-1/2}K_{\lambda-1/2}(\alpha\sqrt{\delta^2+(z-\mu)^2})e^{\beta(z-\mu)} \tag{4}$$

where $a_\lambda(\alpha,\beta,\delta)$ is a normalizing constant of the form

$$a_\lambda(\alpha,\beta,\delta) = \frac{(\sqrt{\alpha^2-\beta^2})^\lambda}{\sqrt{2\pi}\delta^\lambda\alpha^{\lambda-1/2}K_\lambda(\delta\sqrt{\alpha^2-\beta^2})}. \tag{5}$$

To get NIG, we simply let $\lambda = -\frac{1}{2}$ above with the restrictions $\delta > 0$, $0 \leq |\beta| \leq \alpha$ and $\mu \in R$. The parameters $\alpha, \beta, \delta, \mu$ play different roles. α determines how flat the density function is. It takes on positive values. β determines the skewness of the distribution. If $\beta = 0$, we get a symmetric distribution. δ corresponds to the scale of the distribution while μ is responsible for the shift of the probability density function.

The probability density function of $NIG(\alpha,\beta,\delta,\mu)$ looks complicated but it has a simple moment generating function of the form

$$M_Z(t) = exp\{t\mu + \delta(\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+t)^2})\} \tag{6}$$

from which we get the mean, variance, skewness and kurtosis as $\mu+\delta\frac{\beta}{\sqrt{\alpha^2-\beta^2}}$, $\delta\frac{\alpha^2}{\{\sqrt{\alpha^2-\beta^2}\}^3}$, $\frac{3\beta}{\alpha\sqrt{\delta\sqrt{\alpha^2-\beta^2}}}$ and $3\frac{(1+4(\frac{\beta^2}{\alpha^2}))}{\delta\sqrt{\alpha^2-\beta^2}}$ respectively.

2.3 More NIG useful properties for closing price modeling

1. If $Z \sim NIG(\alpha,\beta,\delta,\mu)$, then $Y = kZ \sim (\alpha/k, \beta/k, \delta/k, \mu/k)$.
2. If $Z_1 \sim NIG(\alpha,\beta,\delta_1,\mu_1)$ and $Z_2 \sim NIG(\alpha,\beta,\delta_2,\mu_2)$ are independent, then the sum $Y = Z_1+Z_2 \sim NIG(\alpha,\beta,\delta_1+\delta_2,\mu_1+\mu_2)$.
3. If $Z_i \sim NIG(\alpha,\beta,\delta,\mu)$, $(i = 1,2,\cdots,n)$ are independent, then the sample mean $\overline{Z} = \frac{1}{n}\sum_{i=1}^n Z_i \sim NIG(n\alpha, n\beta, \delta, \mu)$.
4. If $Z \sim NIG(\alpha,\beta,\delta,\mu)$, then the variable $Y = (Z-\mu)/\delta \sim NIG(\alpha\delta, \beta\delta, 0, 1)$, the standard NIG distribution.

Property 2 above is unique for NIG distribution.

3 Model selection and analysis

3.1 Käärik and Umbleja (2011) proposed model selection strategy

1. choose a suitable class of distributions (using general or prior information about the specific data) ;
2. estimate the parameters (by finding maximum likelihoods);

[2] It is nice to inject here that applying the Central Limit Theorem (CLT) to a random walk, we arrive at the standard normal distribution.

3. estimate goodness of fit;

    - visual estimation
    - classical goodness-of-fit tests (Kolmogorov-smirnov, chi-squared with equiprobable classes),
    - probability or quantile-quantile plots [6].

3.2 Implementation of proposed strategy and analysis

3.2.1 Data description

To implement the above strategy using NIG distribution, we chose three categories of financial data. The first category comes from the Tallinn Stock exchange, and consist of the closing prices of four companies (Arco Vara, Baltika, Ekpress Grupp and Harju Elekter) trading between 01 January 2008 to 01 January 2012, spanning part of the financial crises period. We could have considered more data points, but this exchange is pretty new. It is also interesting as Estonian economy is relatively small compared with those of Turkey, Britain, Russia, USA etc. Thus this data serves also as a check against data collected from a big economy which may follow already known trends for financial data.

Second data represents daily closing prices of some world indexes as quoted in the USA through yahoo finance. It covers the period from 21 April 2004 to 29 Dec 2011. This is pretty long stretch of time with many data points than the first set of data. This also covers the recent financial bubble. Index prices normally incorporate the underlying stochastic properties of the assets on which they are composed of, thus we have a different kind of financial data. The considered indexes are GSPC, STI, FTSE100 and OMXSPI.

Third data source is from the UK, and these are the quoted daily closing prices of currency trades or foreign exchange (FX) closing prices. This is quite interesting data as its properties are slightly different from those of general financial data. It is known to encompass more volatility and interfered with by governmental policies on a regular basis. Since it covers the recent financial bubble, it is quite interesting as fiscal stimulus decisions by governments are reflected in the prices. The period is from 12 April 2008 to 07 August 2012 and the FX are CHF/JPY, USD/EUR, EUR/GBP, SAR/CHF, QAR/CHF and EGP/CHF.

3.2.2 Analysis

We calculated the skews and kurtoses of all the stock market constituents, and the results, together with estimated NIG parameters (through maximum likelihoods) are displayed in the Table 1. These clearly portray the need for a distribution that can capture tails and has some peaks; an excellent quality of NIG distribution. Goodness of fits tests were with Kolmogorov-Smirnov(KS) and Chi-square(CS). From Table 1, we can see that CS test rejected null hypothesis, while KS had excellent results; but for Arco Vara and Harju Elekter. We were also slightly dissapointed by STI index, even though the results are acceptable. FX rates passed KS test really good. Our conclusions were similar studying Figures 1 to 3. Thus the closing prices of Baltika and Ekpress Grupp (companies trading on Tallinn stock exchange), FTSE100, GSPC and STI (major world indexes), CHF/JPY, USD/EUR, EUR/GBP, SAR/CHF, QAR/CHF and EGP/CHF (Foreign Exchange rates) can be modeled by NIG distribution.

Name	alpha(α)	beta(β)	delta(δ)	mu(μ)	Skew	Kurtosis	$\chi^2 stat$	$\chi^2 - p$	KS d	KS p
Arco Vara	468.9	468.86	0.03	0.02	0.38	-1.53	2251.60	$<10^{-5}$	0.23	$<10^{-5}$
Baltika	7.06	6.62	0.22	0.52	1.67	-1.53	1771.12	$<10^{-5}$	0.06	0.06
Ekpress	2.68	2.15	0.49	0.85	1.70	2.53	1194.24	$<10^{-5}$	0.07	0.012
Harju	3.20	-2.07	0.72	2.95	-0.82	-0.05	1345.87	$<10^{-5}$	0.09	0.0003
FTSE100	0.03	-0.028	1195.17	7947.69	-3.92	-5.5	82.62	1	0.04	0.097
GSPC	0.04	-0.03	514.04	1667.66	-0.05	0.12	36.49	1	0.015	0.016
OMXSPI	0.93	-0.70	870.55	1311.78	-0.06	-0.76	122.57	1	0.06	0.002
STI	0.93	-0.7	870.55	1311.78	-0.06	-0.76	122.57	1	0.053	0.0096
CHF/JPY	0.52	0.24	7.21	82.62	0.72	1.16	140.7	1	0.037	0.5
QAR/CHF	2142.72	-2082	0.0195	0.366	-0.78	0.3	344.31	1	0.06	0.06
USD/EUR	843.07	744.8	0.19	0.358	0.19	-0.6	208.12	1	0.043	0.31
EGP/CHF	177.87	-146.89	0.02	0.21	-0.995	0.17	528.07	0.1	0.06	0.06
EUR/GBP	3742.17	-1392.14	3.46	2.25	-0.01	-0.07	215.02	1	0.045	0.26
SAR/CHF	2665.64	-2593.4	0.018	0.356	-0.77	0.29	325.68	1	0.047	0.22

Table 1: Estimated NIG Parameters, Skews, Kurtoses, Chi-square(χ^2) and Kolmogorov-Smirnov(KS) test results

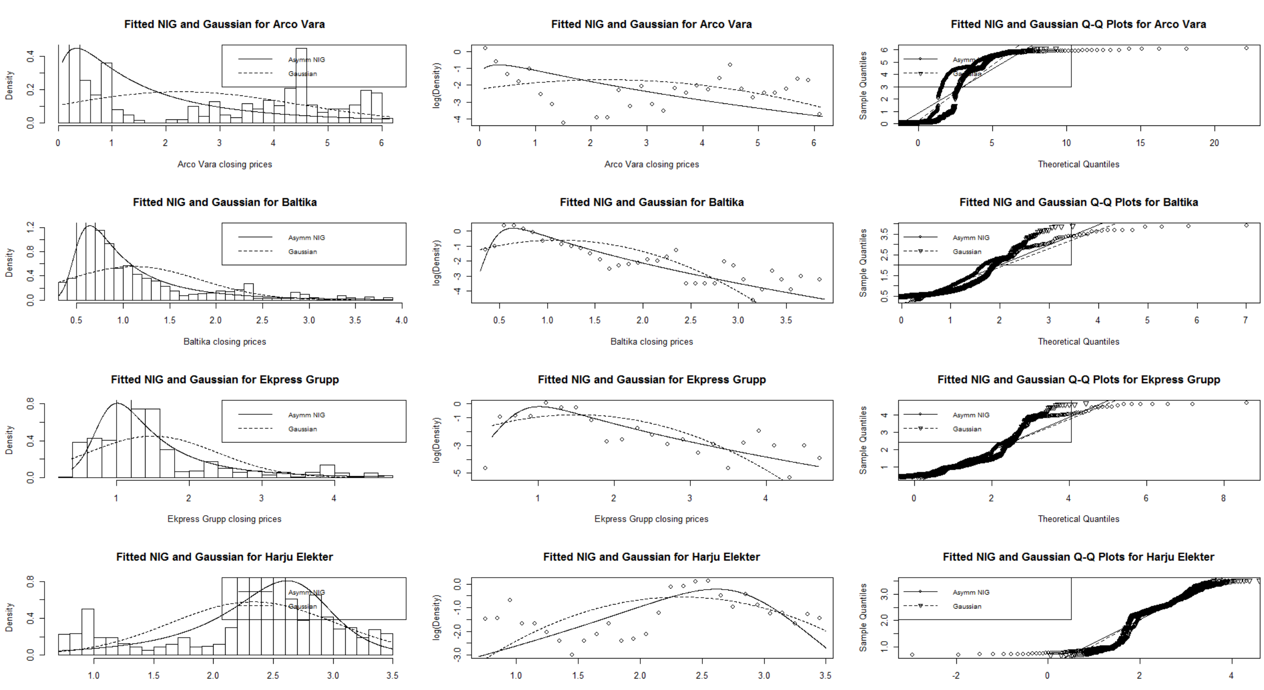

Figure 1: Fitted Gaussian and NIG densities, log densities and Q-Q plots for Baltika, Arco Vara, Harju Elekter and Ekpress Grupp

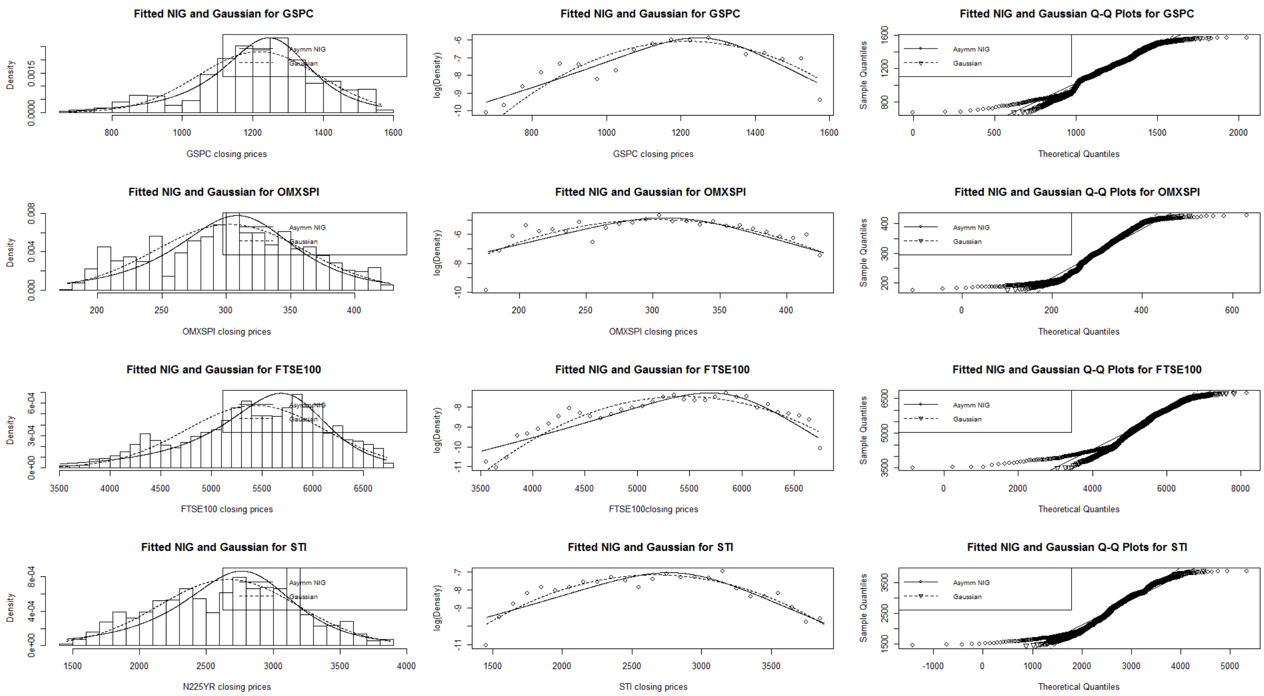

Figure 2: Fitted Gaussian and NIG densities, log densities and Q-Q plots for GSPC, STI, FTSE100 and OMXSPI indexes

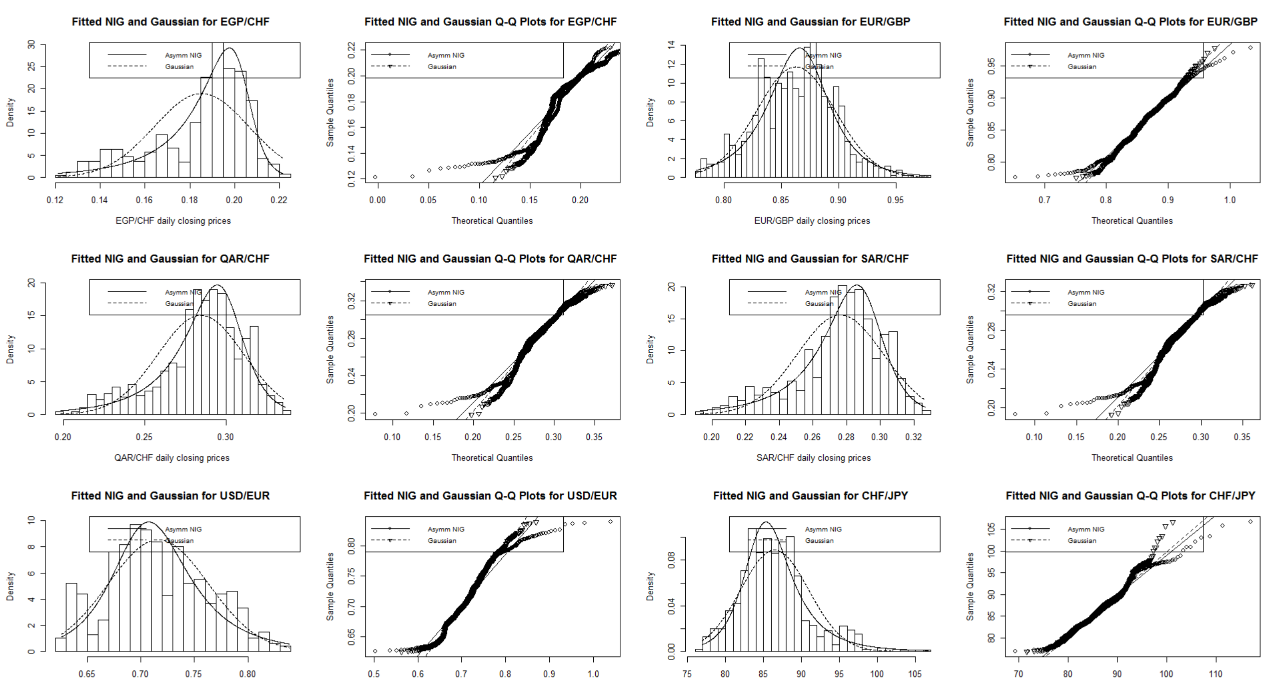

Figure 3: Fitted Gaussian and NIG densities and Q-Q plots for NIG FX models

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