Compact and Fredholm Operators on Matrix Domains of Triangles in the Space of Null Sequences

E. Malkowsky

Department of Mathematics, Fatih University, Istanbul, Turkey

Department of Mathematics, University of Giessen, Giessen, Germany

Abstract The matrix domain X_A of an infinite matrix $A = (a_{nk})_{n,k=0}^{\infty}$ of complex numbers in a subset X of the set ω of all complex sequences is the set of all $x = (x_k)_{k=0}^{\infty} \in \omega$ for which the series $A_n x = \sum_{k=0}^{\infty} a_{nk} x_k$ converge for all n and $Ax = (A_n x)_{n=0}^{\infty} \in X$. Also, if X and Y are subsets of ω then (X, Y) denotes the set of all infinite matrices that map X into Y, that is, $A \in (X, Y)$ if and only if $X \subset Y_A$. Let c_0 denote the set sequences $x \in \omega$ that converge to zero, and $T = (t_{nk})_{n,k=0}^{\infty}$ and $\tilde{T} = (\tilde{t}_{nk})_{k,k=0}^{\infty}$ be triangles, that is, $t_{nk} = \tilde{t}_{nk} = 0$ for $k > n$ and $t_{nn} = \tilde{t}_{nn} \neq 0$ $(n = 0, 1, \dots)$. We characterise the class $((c_0)_T, (c_0)_{\tilde{T}})$. Furthermore we obtain an explicit formula for the Hausdorff measure of noncompactness of operators L_A given by a matrix $A \in (c_0)_T, (c_0)_{\tilde{T}})$, that is, for which $L_A(x) = Ax$ for all $x \in (c_0)_T$. From this result, we obtain a characterisation the class of compact operators given by matrices in $((c_0)_T, (c_0)_{\tilde{T}})$. Finally we give a sufficient condition for an operator given by a matrix to be a Fredholm operator on $(c_0)_T$.

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