A Third-Order of Accuracy Difference Scheme for the Bitsadze-Samarskii Type Nonlocal Boundary Value Problem

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Abstract

The role played by coercive inequalities in the study of local boundary-value problems for elliptic and parabolic differential equations is well-known ([1], [2]). Theory, applications and methods of solutions of Bitsadze-Samarskii nonlocal boundary value problems for elliptic differential equations have been studied extensively by many researchers ([3]-[5]). The Bitsadze-Samarskii type nonlocal boundary value problem

$$\begin{cases} -\frac{d^2u(t)}{dt^2} + Au(t) = f(t),\ 0 < t < 1, \\ \\ u_t(0) = \varphi, u_t(1) = \beta u_t(\lambda) + \psi, \\ \\ 0 \le \lambda < 1,\ |\beta| \le 1 \end{cases}$$

for the differential equation in a Hilbert space H with the self-adjoint positive definite operator A is considered. The third order of accuracy difference scheme for the approximate solution of this problem is presented. The well-posedness of this difference scheme in difference analogue of Hölder spaces is established. In applications, the stability, the almost coercivity and the coercivity estimates for solution of difference scheme for elliptic equations are obtained.

References

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