Sturm Liouville Problem with Discontinuity Conditions at Several Points

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Abstract

In this paper we deal with the computation of the eigenvalues of Sturm Liouville problem with several discontinuity conditions (transmission conditions) inside a finite interval and parameter dependent boundary conditons.By using an operator theoretic interpretation we extend some classic results for regular Sturm Liouville problems.A symmetric linear operator A is defined in an appropriate Hilbert space such that the eigenvalues of such a problem coincide with those of A. Also,we obtained asymptotic formulaes for the eigenvalues and corresponding eigenfunctions.

Consider the following Sturm Liouville problem with discontinuity conditions at several points inside a finite interval,

$$\tau(u) := -u'' + q(x)u = \lambda u, \ x \in (x_0, x_1) \tag{1}$$

$$B_1(u) := \beta_1 u(x_0) + \beta_2 u'(x_0) = 0 \tag{2}$$

$$B_2(u) := \lambda(\alpha_1' u(x_1) - \alpha_2' u'(x_1)) + \alpha_1 u(x_1) - \alpha_2 u'(x_1) = 0 \tag{3}$$

$$T_k(u) := \begin{pmatrix} u(\theta_k + 0) \\ u'(\theta_k + 0) \end{pmatrix} - D_k \begin{pmatrix} u(\theta_k - 0) \\ u'(\theta_k - 0) \end{pmatrix} = 0, \quad k = \overline{1, m} \tag{4}$$

where $x_0 = \theta_0 < \theta_1 < ... < \theta_m < \theta_{m+1} = x_1$, $q \in L_2(x_0, x_1)$, λ is a complex spectral parameter. We shall assume that $\beta_1^2 + \beta_2^2 \neq 0, \alpha_1^2 + \alpha_2^2 \neq 0, \rho > 0$,where $\rho = \begin{cases} \infty & \text{, if } \alpha_1' + \alpha_2' = 0 \\ \alpha_1'\alpha_2 - \alpha_2'\alpha_1 & \text{, otherwise} \end{cases}$ and $D_k = \begin{pmatrix} \gamma_{1k} & \gamma_{2k} \\ \gamma_{3k} & \gamma_{4k} \end{pmatrix}$, $\gamma_{ik} \in \mathbb{R}$, $i = \overline{1,4}$, $|D_k| > 0$ for $k = \overline{1, m}$. Let D_0 be the 2×2 identity matrix.

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