On Bitsadze-Samarskii type nonlocal boundary value problems for semilinear elliptic equations

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Abstract

In the literature, the problem of Bitsadze-Samarskii type is often referred to as the boundary value problem with Bitsadze-Samarskii condition (see [2], [4] and [7]). Previously, the Bitsadze-Samarskii type nonlocal boundary value problems for linear elliptic equations were studied ([5]). In this paper, the Bitsadze-Samarskii type nonlocal boundary value problems for semilinear elliptic equations

$$\begin{cases} -\frac{d^2u(t)}{dt^2} + Au(t) = f(t, u(t)), 0 < t < 1, \\ u(0) = \varphi,\ u(1) = \sum\limits_{j=1}^{J} \alpha_j u(\lambda_j) + \psi, \\ 0 < \lambda_1 < \cdots < \lambda_J < 1, \sum\limits_{j=1}^{J} |\alpha_j| \le 1 \end{cases}$$

in a Hilbert space H with the self-adjoint positive definite operator A is considered. The first and second orders of accuracy difference schemes approximately solving these problems are studied. A procedure of modified Gauss elimination method is used for solving these difference schemes for the two-dimensional elliptic differential equation. The method is illustrated by numerical examples. The converge estimates for the solution of these difference schemes are obtained.

References

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