# On the Numerical Solution of Ultra Parabolic Equations with Neumann Condition

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**Abstract**

In this paper, our interest is studying the stability of first order difference scheme for the approximate solution of the initial boundary value problem for ultra parabolic equations

$$\begin{cases} \frac{\partial u(t,s)}{\partial t}+\frac{\partial u(t,s)}{\partial s}+Au(t,s)=f(t,s), \quad 0<t,s<T, \\ u(0,s)=\psi(s), \quad 0\leq s\leq T, \\ u(t,0)=\varphi(t), \quad 0\leq t\leq T \end{cases} \tag{1}$$

in an arbitrary Banach space E with a strongly positive operator A.We refer to [1, 2] and the references therein for a series of papers by the authors, dealing with ultra parabolic equations , arising in diffusion theory, probability and finance. Some new results about numerical methods for ultra-parabolic equations are also announced, see [3-5]. For approximately solving problem (1), the first-order of accuracy difference scheme

$$\begin{cases} \frac{u_{k,m}-u_{k-1,m}}{\tau}+\frac{u_{k-1,m}-u_{k-1,m-1}}{\tau}+Au_{k,m}=f_{k,m} \ , \\ f_{k,m}=f(t_k,s_m),\, t_k=k\tau,\ s_m=m\tau,\ 1\leq k,m\leq N,\ N\tau=1, \\ u_{0,m}=\psi_m,\psi_m=\psi(s_m),\ 0\leq m\leq N, \\ u_{k,0}=\varphi_k,\varphi_k=\varphi(t_k),\ 0\leq k\leq N \end{cases} \tag{2}$$

is presented. The stability estimates and almost coercive stability estimates for the solution of difference schemes (2) is established. In applications, the stability in maximum norm of difference shemes for multidimensional ultra parabolic equations with Neumann condition is established. Applying the difference schemes, the numerical methods are proposed for solving one dimensional ultra parabolic equations.

**References**

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