Reproducing Kernel Hilbert Space Method for Solving the Pollution Problem of Lakes

Z. Karabulut[1] and V. S. Ertürk[2]

[1]Department of Mathematics, Ondokuz Mayıs University, Samsun, Turkey

[2]Department of Matehmatics,Ondokuz Mayıs University, Samsun, Turkey

Abstract

Pollution is a major threat for our environment. Monitoring pollution is the first step to save environment and has become possible with use of differential equations. This study includes the problem of pollution of three lakes connected with pipes or channels [4]. Consider the following mathematical model describing the pollution of a system of lakes [1-3] :

$$\begin{cases} \frac{dx_1}{dt} = \frac{F_{13}}{V_3}x_3(t) + p(t) - \frac{F_{31}}{V_1}x_1(t) - \frac{F_{21}}{V_1}x_1(t) \\ \frac{dx_2}{dt} = \frac{F_{21}}{V_1}x_1(t) - \frac{F_{32}}{V_2}x_2(t) \\ \frac{dx_3}{dt} = \frac{F_{31}}{V_1}x_1(t) + \frac{F_{32}}{V_2}x_2(t) - \frac{F_{13}}{V_3}x_3(t) \end{cases} \quad (1)$$

The approximate solutions are obtained with Reproducing Kernel Hilbert Space Method [5-6] for three different models: impulse, step and sinusoidal. The absolute errors are calculated by comparing the numerical results to the analytic results. The errors are seen to be acceptable. All of the numerical computations have been calculated on a computer programme with MATHEMATICA .

References

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