Radial Basis Functions Method for determining of unknown coefficient in parabolic equation

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Abstract

In this paper, we consider an inverse problem of finding unknown source parameter$p(t)$ and $u(x,t)$ satisfy equation

$$u_t = u_{xx} + p(t)u + f\left(t,x\right), \qquad 0 \leqslant x \leqslant 1,\ 0 < t \leqslant T, \tag{1}$$

with the initial-boundary conditions

$$u(x,0) = \varphi(x), \qquad 0 \leqslant x \leqslant 1 \tag{2}$$

$$(0,t) = \mu_1(t), \qquad 0 < t \leqslant T \tag{3}$$

$$u(1,t) = \mu_2(t), \qquad 0 < t \leqslant T \tag{4}$$

subject to the overspecification over the spatial domain

$$u(x^*,t) = E(t), \qquad 0 < x^* \leqslant 1,\ 0 < \ t \leqslant T \tag{5}$$

where $f(x,t), \varphi(x), \mu_1(t), \mu_2(t)$ and $E(t) \neq 0$ are known functions, x^* is a fixed prescribed interior point in (0,1). If $p\left(t\right)$ is known then direct initial boundary value problem $(1)-(4)$ has a unique smooth solution $u\left(x,t\right)$ [1]. If u represent a temperature distribution, then $(1) - (4)$ can be interpreted as a control problem with source parameter. Based on the idea of the radial basis functions (RBF) approximation , a fast and highly accurate meshless method is developed for solving an inverse problem with a control parameter [2]. Some numerical examples using the proposed algorithm are presented.

References

[1] Isakov V., Inverse Problems for Partial Differential Equations, Applied Mathematical Sciences, Springer-Verlag, vol. 127, 1997.

[2] Limin Ma and Zongmin Wu, Radial Basis functions method for parabolic inverse problem, Int. J. of Computer Math., 88(2), 383-395, 2011.