On the Solution of a Three Dimensional Convection Diffusion Problem

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Abstract

In this paper, the Rothe difference scheme and the Adomian Decomposition method are presented for obtaining the approximate solution of three dimensional convection-diffusion problem. Stability estimates for the difference problem is presented.

Keywords: Finite difference method, Adomian Decomposition Method, Convection-diffusion equation

1 Introduction

In many important applications in engineering such as transport of air and water pollutants, convection-diffusion problems arises. An example of this kind of problem is a forced heat transfer. Several numerical methods are proposed for solving three dimensional convection diffusion problem (see [1]-[11] and the references therein). In this paper, we focus on the following mixed problem for the three dimensional convection-diffusion equation

$$\begin{cases} \frac{\partial u}{\partial t} + b_1(x,y,z)\frac{\partial u}{\partial x} + b_2(x,y,z)\frac{\partial u}{\partial y} + b_3(x,y,z)\frac{\partial u}{\partial z} \\ -\left(a_1\frac{\partial^2 u}{\partial x^2} + a_2\frac{\partial^2 u}{\partial y^2} + a_3\frac{\partial^2 u}{\partial z^2}\right) = f(t,x,y,z), \text{ in } \Omega \times P, \\ u(x,y,z,t) = 0, \text{ on } \partial\Omega \times \overline{P}, \\ u(x,y,z,0) = g(x,y,z), \text{ in } \overline{\Omega}, \end{cases} \quad (1)$$

where $\Omega = (0,1)\times(0,1)\times(0,1)$, $P = (0,T), b_1(x,y,z), b_2(x,y,z), b_3(x,y,z), g(x,y,z)$ are sufficiently smooth functions and a_1, a_2, a_3 are positive constants. Here, $b_1(x,y,z)$, $b_2(x,y,z)$, $b_3(x,y,z)$, a_1, a_2 and a_3 are velocity components of the fluid in the directions of the axes at the point (x,y,z) at time t.

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