Using Inverse Laplace Transform for the solution of a Flood Routing Problem

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Abstract

The inverse Laplace transform is of significant importance in mathematical sciences when an analytical solution exists in Laplace domain. So, a new solution of the linearized St. Venant equations (LSVE) has been obtained for flood routing in open channels. The LSVE has been previously used by many researchers [1] and in Laplace domain are in the matrix form

$$\frac{\partial \kappa}{\partial t} + A\frac{\partial \kappa}{\partial x} + B\kappa = 0 \qquad (1)$$

where κ is transfer matrix includes deviations of discharge $q(x,t)$ and depth $y(x,t)$ around the reference values (Q_0, Y_0). In this new formulation, the Manning formula is linearized as boundary condition besides the St. Venant equations to get a Laplace transformable, simplified set of equations in Laplace domain as follows

$$\hat{q}(L,s) = k_v \hat{y}(L,s) \qquad (2)$$

where $k_v = \frac{\partial Q}{\partial Y}$. A method for Laplace inversion, which provides a great convergence, very accurate response for flood routing problem is used here. As previously this method has been used for difuusion waves model [2, 3], the results show the improved De Hoog algorithm [4] provide a solution with zero error for discharge, and very small percent of error for depth. Applying the well-known Preissmann implicit scheme on the LSVE for equal condition shows that the De Hoog algorithm is in complete agreement with the numerical solution of the LSVE.

References

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