## **Multibody Railway Vehicle Dynamics Using Symbolic Mathematics**

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## Abstract

In this work, the Equations of Motion (EOMs) of the Multibody Dynamics is derived for a railway vehicle. The previous work of the authors is related to derive the Multibody Dynamics model of the bogie with 44 DAEs (see [1]). Lagrange dynamics is used as common approach in applied mathematics and mechanics for computational differential-algebraic equations (CDAEs). Differential equations of motions are formulized as in the generalized symbolic mathematics and applied in the Matlab's MuPad Symbolic Math Toolbox.

The size of the railway vehicle's DAEs is about 156. Finally, the results are compared using eigenvalues with previous studies in the same area with a success. The symbolic mathematics is currently used for derivation of the multibody dynamics EOMs (see [2] and [4]). Langrange dynamics for the trajectory coordinate is applied to derive generalized EOMs for the multibody dynamics. Following Equation 1 is one of the generalized equation used to derive the state space representation of the EOMs for the railway vehicle (see [3]).

$$\begin{cases} \dot{\mathbf{p}}^i \\ \ddot{\mathbf{p}}^i \end{cases} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1} & \mathbf{K} & -\mathbf{M}^{-1} & \mathbf{C} \end{bmatrix} \begin{cases} \mathbf{p}^i \\ \dot{\mathbf{p}}^i \end{cases} + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix} \mathbf{u} + \mathbf{E}$$

$$(1)$$

**Keywords**: Computational differential-algebraic equations (CDAEs), Multibody dynamics (MBD), Eigenvalue analysis, Lagrange dynamics, Railway vehicles.

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## References

[1] H. Sahin , A. Kar, and E. Tacgin, "Analysis of the Differential-Algebraic Equations (DAEs) for Multibody Dynamics", *Proceedings of the International Conference on Applied Analysis and Algebra*, Istanbul, Turkiye, June 20-24, 2012.

[2] T. Kurz, P. Eberhard., & S. C. Henninger, From Neweul to Neweul-M2: symbolic equations of motion for multibody system analysis and synthesis. *Multibody System Dynamics*, 2010, 25-41.

[3] Rieveley, R. J. The Effect of Direct Yaw Moment on Human Controlled Vehicle Systems. *Dissertation*. Windsor, Ontario, Canada: University of Windsor, 2010, May 19.

[4] C. Smitke and P. Goossens, "Symbolic Computation Techniques for Multibody Model Development and Code Generation" in *ECCOMAS Thematic Conference on Multibody Dynamics*, Brussels, Belgium, 4-7 July 2011.