

On Hadamard Type Integral Inequalities For Nonconvex Functions

Mehmet Zeki Sarikaya¹, Hakan Bozkurt¹ and Necmettin Alp¹

¹Department of Mathematics, Duzce University, Duzce, Turkey

Abstract

Convexity plays a central and fundamental role in mathematical finance, economics, engineering, management sciences and optimization theory. In recent years, several extensions and generalizations have been considered for classical convexity. A significant generalization of convex functions is that of φ -convex functions introduced by Noor in [3]. In [3] and [6], the authors have studied the basic properties of the φ -convex functions. It is well-known that the φ -convex functions and φ -sets may not be convex functions and convex sets. This class of nonconvex functions include the classical convex functions and its various classes as special cases. For some recent results related to this nonconvex functions, see the papers [3]-[6]. In this article, using functions whose derivatives absolute values are φ -convex and quasi- φ -convex, we obtained new inequalities related to the right and left side of Hermite-Hadamard inequality. In particular if $\varphi = 0$ is taken as, our results obtained reduce to the Hermite-Hadamard type inequality for classical convex functions.

References

- [1] S.S. Dragomir and R.P. Agarwal, *Two inequalities for differentiable mappings and applications to special means of real numbers and trapezoidal formula*, Appl. Math. Lett., 11(5) (1998), 91–95.
- [2] S. S. Dragomir and C. E. M. Pearce, *Selected Topics on Hermite-Hadamard Inequalities and Applications*, RGMIA Monographs, Victoria University, 2000.
- [3] M. Aslam Noor, *Some new classes of nonconvex functions*, Nonl.Funct.Anal.Appl.,11(2006),165-171
- [4] M. Aslam Noor, *On Hadamard integral inequalities involving two log-preinvex functions*, J. Inequal. Pure Appl. Math., 8(2007), No. 3, 1-6, Article 75.
- [5] M. Aslam Noor, *Hermite-Hadamard integral inequalities for log- φ – convex functions*, Nonl. Anal. Forum, (2009).
- [6] M. Aslam Noor, *On a class of general variational inequalities*, J. Adv. Math. Studies, 1(2008), 31-42.
- [7] K. Inayat Noor and M. Aslam Noor, *Relaxed strongly nonconvex functions*, Appl. Math. E-Notes, 6(2006), 259-267.
- [8] U.S. Kırmacı, *Inequalities for differentiable mappings and applications to special means of real numbers and to midpoint formula*, Appl. Math. Comp., 147 (2004), 137-146.
- [9] U.S. Kırmacı and M.E. Özdemir, *On some inequalities for differentiable mappings and applications to special means of real numbers and to midpoint formula*, Appl. Math. Comp., 153, (2004), 361-368.
- [10] U.S. Kırmacı, *Improvement and further generalization of inequalities for differentiable mappings and applications*, Computers and Math. with Appl., 55 (2008), 485-493.
- [11] D.A. Ion, *Some estimates on the Hermite-Hadamard inequality through quasi-convex functions*, Annals of University of Craiova Math. Comp. Sci. Ser., 34 (2007) 82-87.
- [12] C.E.M. Pearce and J. Pečarić, *Inequalities for differentiable mappings with application to special means and quadrature formulae*, Appl. Math. Lett., 13(2) (2000), 51–55.

- [13] J. Pečarić, F. Proschan and Y.L. Tong, *Convex functions, partial ordering and statistical applications*, Academic Press, New York, 1991.
- [14] M. Z. Sarikaya, A. Saglam and H. Yildirim, *New inequalities of Hermite-Hadamard type for functions whose second derivatives absolute values are convex and quasi-convex*, International Journal of Open Problems in Computer Science and Mathematics (IJOPCM), 5(3), 2012.
- [15] M. Z. Sarikaya, A. Saglam and H. Yildirim, *On some Hadamard-type inequalities for h-convex functions*, Journal of Mathematical Inequalities, Volume 2, Number 3 (2008), 335-341.
- [16] M. Z. Sarikaya, M. Avci and H. Kavurmaci, *On some inequalities of Hermite-Hadamard type for convex functions*, ICMS International Conference on Mathematical Science. AIP Conference Proceedings 1309, 852 (2010).
- [17] M. Z. Sarikaya and N. Aktan, *On the generalization some integral inequalities and their applications* Mathematical and Computer Modelling, Volume 54, Issues 9-10, November 2011, Pages 2175-2182.
- [18] M. Z. Sarikaya, E. Set and M. E. Ozdemir, *On some new inequalities of Hadamard type involving h-convex functions*, Acta Mathematica Universitatis Comenianae, Vol. LXXIX, 2(2010), pp. 265-272.
- [19] A. Saglam, M. Z. Sarikaya and H. Yildirim, *Some new inequalities of Hermite-Hadamard's type*, Kyungpook Mathematical Journal, 50(2010), 399-410.
-