

On Hermite-Hadamard Type Integral Inequalities for preinvex and log-preinvex functions

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Abstract

It is well known that convexity has been playing a key role in mathematical programming, engineering, and optimization theory. The generalization of convexity is one of the most important aspects in mathematical programming and optimization theory. There have been many attempts to weaken the convexity assumptions in the literature, (see, [1], [2], [8]-[11], [16]-[21]). A significant generalization of convex functions is that of invex functions introduced by Hanson in [12]. Ben-Israel and Mond [14] introduced the concept of preinvex functions, which is a special case of invexity. Pini [15] introduced the concept of prequasiinvex functions as a generalization of invex functions. Noor [5]-[7] has established some Hermite-Hadamard type inequalities for preinvex and log-preinvex functions. In recent papers Barani, Ghazanfari, and Dragomir in [3] presented some estimates of the right hand side of a Hermite-Hadamard type inequality in which some preinvex functions are involved. His class of nonconvex functions include the classical convex functions and its various classes as special cases. For some recent results related to this nonconvex functions, see the papers ([4]-[7], [12]-[15]). In this paper, we extend some estimates of the left hand side of a Hermite-Hadamard type inequality for nonconvex functions whose derivatives absolute values are preinvex and log-preinvex.

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