

Fractional Parabolic Differential and Difference Equations with the Dirichlet-Neumann Condition

Allaberen Ashyralyev^{*,†}, Nazar Emirov^{**} and Zafer Cakir[‡]

^{*}Department of Mathematics Fatih University, Buyukcekmece 34500, Istanbul, Turkey, aashyr@fatih.edu.tr,

[†]Department of Mathematics, ITTU, Ashgabat, Turkmenistan

^{**}Department of Mathematics Fatih University, 34500 Buyukcekmece, Istanbul, Turkey,
nazaremirov@gmail.com

[‡]Department of Mathematical Engineering, Gumushane University, 29100, Gumushane, Turkey,
zafer@gumushane.edu.tr

Abstract. The multidimensional fractional parabolic equation with the Dirichlet-Neumann condition is studied. Stability estimates for the solution of the initial-boundary value problem for this fractional parabolic equation are established. The stable difference schemes for this problem are presented. Stability estimates for the solution of the first order of accuracy difference scheme are obtained. A procedure of modified Gauss elimination method is applied for the solution of first and second order of accuracy difference schemes of one-dimensional fractional parabolic differential equations.

Keywords: Fractional parabolic equations, Dirichlet-Neumann condition, Positive operator, Difference schemes, Stability

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