NBVP for Hyperbolic Equations Involving Multi-point and Integral Conditions

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Abstract

Nonlocal boundary value problems involving multi-point and integral conditions for a hyperbolic equation in a Hilbert space are investigated. The stability estimates for the solution of these

multi-point NBVP are established. In applications, the stability estimates for the solution of these problems are obtained.

The authors of [3] developed a numerical procedure for the NBVP with a integral conditions for hyperbolic equations. In the paper [4], instead of nonlocal integral conditions multi-point nonlocal conditions used. In the present work, we consider the NBVP with multi-point and integral conditions

$$\begin{cases} \frac{d^2u(t)}{dt^2} + Au(t) = f(t) \quad (0 \leq t \leq 1), \\ u(0) = \int\limits_0^1 \alpha(\rho) u(\rho) d\rho + \sum\limits_{i=1}^{n} a_i u(\lambda_i) + \varphi, \\ u_t(0) = \int\limits_0^1 \beta(\rho) u_t(\rho) d\rho + \sum\limits_{i=1}^{n} b_i u_t(\lambda_i) + \psi \end{cases} \tag{1}$$

for the differential equation in a Hilbert space H with a self-adjoint positive definite operator A. We are interested in studying the stability of solutions of problem (1) under the assumption

$$\left| 1 + \int\limits_0^1 \alpha(s)\beta(s)\,ds + \sum_{k=1}^{n} a_k b_k + \sum_{k=1}^{n} a_k \int\limits_0^1 \beta(s)\,ds + \sum_{k=1}^{n} b_k \int\limits_0^1 \alpha(s) ds \right| \tag{2}$$

$$> \int\limits_0^1 (|\alpha(s)| + |\beta(s)|)\,ds + \sum_{k=1}^{n} |a_k + b_k|.$$

A function $u(t)$ is a *solution* of problem (1) if the following conditions are satisfied:

i) $u(t)$ is twice continuously differentiable on the interval $(0,1)$ and continuously differentiable on the segment $[0,1]$. The derivatives at the endpoints of the segment are understood as the appropriate unilateral derivatives.

ii) The element $u(t)$ belongs to $D(A)$ for all $t \in [0,1]$, and function $Au(t)$ is continuous on the segment $[0,1]$.

iii) $u(t)$ satisfies the equation and nonlocal boundary conditions (1).

References

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