Numerical solution of a time-fractional Navier–Stokes Equation with modified Riemann-Liouville derivative

Mehmet Merdan[1], Ahmet Gökdoğan[1]

[1]Gümüşhane University, Department of Mathematical Engineering,
29100-Gümüşhane, Turkey

Abstract

In this paper, fractional variational iteration method (FVIM) is implemented to give an approximate analytical solution of a time-fractional Navier–Stokes Equation. Fractional derivatives are described in the Riemann-Liouville derivative. A new application of fractional variational iteration method (FVIM) was extended to derive analytical solutions in the form of a series for these equations. By using an initial value, the explicit solution of the equation has been presented in the closed form and then its numerical solution has been showed graphically.The behavior of the solutions and the effects of different values of fractional order α are indicated graphically. The results obtained by the FVIM reveal that the method is performs extremely well in terms of efficiency and simplicity method for nonlinear differential equations with modified Riemann-Liouville derivative.

Keywords: Fractional variational iteration method, A time-fractional Navier–Stokes Equation, Riemann-Liouville derivative, Fractional calculus

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