Derivation and numerical study of relativistic Burgers equations posed on Schwarzschild spacetime

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We consider nonlinear hyperbolic balance laws posed on a curved spacetime endowed with a volume form and identify a unique (up to normalization) hyperbolic balance law that enjoys the Lorentz invariance property also shared by the Euler equations of relativistic compressible fluids. The proposed model can be viewed as a relativistic version of Burgers equation and provides us with a simplified model on which numerical methods for hyperbolic equations can be developed and analyzed. This model is also compared with a second model derived directly from the relativistic Euler equations. We then introduce a finite volume scheme for the approximation of discontinuous solutions to the Burgerstype model when the background is chosen to be (a subset of) the Schwarzschild spacetime. Our scheme is formulated geometrically and is consistent with the natural divergence form of the balance law and applies to weak solutions containing shock waves. Most importantly, our scheme is well-balanced in the sense that it preserves static equilibrium solutions. Numerical experiments demonstrate the convergence of the proposed finite volume scheme and its relevance for computing late-time asymptotics of (possibly) discontinuous solutions on a curved background.

This presentation is based on the joint paper [2].

References

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