Optimal Control Problem for Impulsive Systems with Integral Boundary Conditions

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Abstract

Impulsive differential equations have become important in recent years as mathematical models of phenomena in both physical and social sciences. There is a significant development in impulsive theory especially in the area of impulsive differential equations with fixed moments; see for instance the monographs [1-4] and the references therein.

Many of the physical systems can be described better by integral boundary conditions. Integral boundary conditions are encountered in various applications such as population dynamics, blood flow models, chemical engineering and cellular systems. Moreover, boundary value problems with integral conditions constitute a very interesting and important class of problems. They include two, three, multi and nonlocal boundary value problems as special cases, (see [5-7]). For boundary value problems with nonlocal boundary conditions and comments on their importance, we refer the reader to the papers [8-10] and the references therein.

In this paper the optimal control problem is considered, when the state of the system is described by the impulsive differential equations with integral boundary conditions. By the help of the Banach contraction principle the existence and uniqueness of solution is proved for the corresponding boundary problem by the fixed admissible control. The first and the second variation of the functional is calculated. Various necessary conditions of optimality of the first and the second order are obtained by the help of the variation of the controls.

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