# Positivity of Two-dimensional Elliptic Differential Operators in Hölder Spaces

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**Abstract**

This paper considers the following operator

$$Au(t,x) = -a_{11}(t,x)u_{tt}(t,x) - a_{22}(t,x)u_{xx}(t,x) + \sigma u(t,x),$$

defined over the region $\mathbb{R}^+ \times \mathbb{R}$ with the boundary condition $u(0,x) = 0$, $x \in \mathbb{R}$. Here, the coefficients $a_{ii}(t,x)$, $i = 1,2$ are continuously differentiable and satisfy the uniform ellipticity

$$a_{11}^2(t,x) + a_{22}^2(t,x) \geq \delta > 0,$$

and $\sigma > 0$. It investigates the structure of the fractional spaces generated by this operator. Moreover, the positivity of the operator in Hölder spaces is proved.

**References**

[1] Krein S.G., Linear Differential Equations in a Banach Space, Amer. Math. Soc., Providence RI, 1968.

[2] Grisvard P., Elliptic Problems in Nonsmooth Domains, Patman Adv. Publ. Program, London, 1984.

[3] Fattorini H.O., Second Order Linear Differential Equations in Banach Spaces, North-Holland: Mathematics Studies, 1985.

[4] Solomyak M.Z., Analytic semigroups generated by elliptic operators in $L_p$ spaces, Dokl. Acad. Nauk. SSSR, 127(1) 37–39, 1959 (Russian).

[5] Solomyak M.Z., Estimation of norm of the reseolvent of elliptic operator in $L_p$ spaces, Usp. Mat. Nauk., 15(6) 141–148, 1960 (Russian).

[6] Krasnosel'skii M.A., Zabreiko P.P., Pustyl'nik E.I. and Sobolevskii P.E., Integral Operators in Spaces of Summable Functions, Nauka, Moscow, 1966 (Russian). English transl.: Integral Operators in Spaces of Summable Functions, Noordhoff, Leiden, 1976.

[7] Stewart H.B., Generation of analytic semigroups by strongly elliptic operators under general boundary conditions, Trans. Amer. Math. Soc. 259 299–310, 1980.

[8] Ashyralyev A. and Sobolevskii P.E., Well-Posedness of Parabolic Difference Equations, Birkhauser Verlag, Basel, Boston, Berlin, 1994.

[9] Ashyralyev A. and Sobolevskii P.E., New difference schemes for partial differential equations, Birkhauser Verlag, Basel, Boston, Berlin, 2004.

[10] Sobolevskii P.E., The coercive solvability of difference equations, Dokl. Acad. Nauk. SSSR 201(5) 1063–1066, 1980 (Russian).

[11] Alibekov Kh.A., Investigations in $C$ and $L_p$ of Difference Schemes of High Order Accuracy for Apporoximate Solutions of Multidimensional Parabolic boundary value problems, PhD Thesis, Voronezh State University, Voronezh, 1978 (Russian).

[12] Alibekov Kh.A. and Sobolevskii P.E., Stability of difference schemes for parabolic equations, Dokl. Acad. Nauk SSSR 232(4) 737–740, 1977 (Russian).

[13] Alibekov Kh.A. and Sobolevskii P.E., Stability and convergence of difference schemes of a high order for parabolic differential equations, Ukrain. Math. Zh. 31(6) 627–634, 1979 (Russian).

[14] Ashyralyev A. and Sobolevskii P.E., The linear operator interpolation theory and the stability of the difference schemes, Dokl. Acad. Nauk SSSR 275(6) 1289–1291, 1984 (Russian).

[15] Ashyralyev A., Method of Positive Operators of Investigations of the High Order of Accuracy Difference Schemes for Parabolic and Elliptic Equations, Doctor of Sciences Thesis, Kiev: Inst. of Math. of Acad. Sci. Kiev, 1992 (Russian).

[16] Simirnitskii Yu.A. and Sobolevskii P.E., Positivity of multidimensional difference operators in the $C-$norm, Usp. Mat. Nauk. 36(4) 202–203, 1981 (Russian).

[17] Danelich S.I., Fractional Powers of Positive Difference Operators, PhD Thesis, Voronezh: Voronezh State University 1989 (Russian).

[18] Ashyralyev A. and N. Yaz, On Structure of Fractional Spaces Generated by Positive Operators with the Nonlocal Boundary Value Conditions, in Proceedings of the Conference Differential and Difference Equations and Applications,, Hindawi Publishing Corporation, New York, edited by R.F. Agarwal and K. Perera, 91–101, 2006.