A Note on the Numerical Solution of Fractional Schrödinger Differential Equations

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Abstract

Many different equations are called by fractional Schrödinger differential equation (FSDE) until today. In recent years, the FSDE which is derived from classical Schrödinger differential equation has received more attention. This problem is solved by some numerical methods (see [2]-[5]). However, finite difference method which is a useful tool for investigation of fractional differential equations has not been applied to a FSDE yet. The present paper fills a gap by applying finite difference method to the following multi-dimensional linear FSDE

$$\begin{cases} i\frac{\partial^\alpha u(t,x)}{\partial t^\alpha} - \sum\limits_{r=1}^{m}(a_r(x)u_{x_r})_{x_r} + \delta u(t,x) = f(t,x), \\ 0 < t < 1, x = (x_1, \cdots, x_m) \in \Omega, \\ u(0,x) = 0, \ x \in \overline{\Omega}, \\ u(t,x) = 0, \ x \in S \end{cases} \tag{1}$$

where $0 < \alpha < 1$. Here $a_r(x), \ x \in \Omega$ and $f(t,x)$ $(t \in [0,1], \ x \in \Omega)$ are given smooth functions and $a_r(x) \geq a \geq 0$. First and second orders of accuracy difference schemes are constructed for problem (1). Numerical experiment on a one-dimensional FSDE shows the effectiveness of the difference schemes.

References

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