An error correction method for solving stiff initial value problems based on a cubic C^1-spline collocation method

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Abstract

For solving nonlinear stiff initial value problems, we develop an improved error correction method (IECM) which originates from the error corrected Euler methods (ECEM) recently developed by the authors (see [17, 18]) and reduces the computational cost and further enhances the stability for the ECEM. We use the stabilized cubic C^1-spline collocation method instead of the Chebyshev collocation method used in ECEM for solving the asymptotic linear ODE for the difference between the Euler polygon and the true solution. It is proved that IECM is A-stable, a semi-implicit one-step method, and of order 4 with only one evaluation of the Jacobian at each integration step. Also, we use the iteration process of the Lobatto IIIA method developed by [13] for solving the induced matrix system. It is shown that this iteration process does not require such the nonlinear function evaluation as the implicit method does and hence it reduces the numerical computational cost efficiently. Numerical evidence is provided to support the theoretical results with several stiff problems.

Keywords: Euler polygon, Cubic C^1-spline collocation method, Lobatto IIIA method, Error correction method, Stiff initial value problem

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[1] This work was supported by basic science research program through the National Research Foundation of Korea(NRF) funded by the ministry of education, science and technology (grant number 2011-0029013).

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