The q analogue of the limit case of bernstein type operators

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Abstract In the present paper we introduce a q-analogue of the Bernstein-typeoperators which is defined in cite{4}. We estimate moments,

establish direct theorems and rate of convergence in terms of themodulus of continuity.

In 1997 Philips [1] proposed the following q-analogue of the well-known Bernstein polynomials,which for each positive integer n and $f \in C[0,1]$,are defined as,

$$B_{n,q}(f;x) = \sum_{k=0}^{n} f\left(\frac{[k]}{[n]}\right) p_{nk}(q;x).$$

After Philips several researchers have studied convergence properties of q-Bernstein polynomials $B_{n,q}(f;x)$. We can refer to readers these important searchs in $[12,13,14,15]$.

P.E. Parvanov , B. D. Popov in 1994 mention Bernstein type operators and examined direct theorems and Jackson type inequality and some approximation properties. This motives us to examine and introduce q analogue of Bernstein type operators.

the class of q-Bernstein operators discussed in this paper are given for natural n by

$$\begin{aligned} U_{n,q}(f;x) &= [n-1] \sum_{k=1}^{n} q^{1-k} p_{nk}(q;x) \int_0^1 f(t)\, p_{n-2,k-1}(q;qt)\, d_q t + f(0)\, p_{n,0}(q;x) \\ &= \sum_{k=1}^{n} b_{nk}(q;x)\, p_{n,k}(q;x). \end{aligned} \tag{1}$$

where $p_{n,k}(q;x) = \begin{bmatrix} n \\ k \end{bmatrix} x^k (1-x)_q^{n-k}$ and the quantity $b_{nk}(q;x) = q^{1-k} \int_0^1 [n-1]\, f(t)\, p_{n-2,k-1}(q;qt)\, d_q t$ for $1 \le k \le n$ in the operators $U_{n,q}(f;x)$ takes place of $f\begin{bmatrix} k \\ n \end{bmatrix}$ in $B_{n,q}(f;x)$ the Bernstein polynomials and $b_{nk}(q;x)$ satisfies $b_{n,0}(f) = f(0)$ and $b_{n,n}(f) = f(1)$.

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