Solution of Differential Equations by Perturbation Technique Using any Time Transformation

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Abstract

A perturbation algorithm using any time transformation is introduced. To account for the nonlinear dependence of the function, we exhibit the function *f* of the system in the differential equation. To this end, we introduce the transformation $T_e = f(w,t)\cdot t$, where f is a function that depends on *t* or *w*. The problems are solved with new time transformation: Linear damped vibration equation, classical Duffing equation and damped cubic nonlinear equation. Results of Multiple Scales, Lindstedt Poincare method, new method and numerical solutions are contrasted [1-6].

Solution of Differential equations by perturbation technique using any time transformation. In Direct Perturbation Method, mostly secular terms appear of higher orders of the expansion invalidating the solution. In order the avoid this problem a new time transformation has been proposed in our study.

The new time transformation is defined as,

$$T_e = f(w,t)\cdot t \tag{1}$$

Using the chain rule, we transform the derivate accordingly

$$\begin{aligned} \frac{du}{dt} &= u'(\dot{f}(w,t)t + f(w,t)) \\ \frac{d^2u}{dt^2} &= u''(\dot{f}(w,t)t + f(w,t))^2 + u'(\ddot{f}(w,t)t + 2\dot{f}(w,t)) \end{aligned} \tag{2}$$

So we have obtained a more effective time expression T_e without losing the original time parameter *t* using the function *f*. Thus, speeding up and slowing down control of the time parameter will be available as in Method of Multiple Scales.

In Equation (2) first order time-derivatives according to new time variable T_e appear in second order time derivative expressions according to original time variable. So, we are able to have information about some parameters of nonlinear differential equation, and to interpret the results.

By this new time transformation we have the advantages of both Lindstedtpoincare method and Method Multi Scales .

Using a new perturbation algorithm with new time transformation, we showed that, first we have obtained a more effective time expression without losing the original time parameter *t* using the function *f*. We are able to have information about some parameters of nonlinear differential equation, and to interpret the results. When we apply this transformation on the known Duffing equation with the results of the studies conducted to date have compared the results obtained.

We found in this new time with the transformation of the solutions are compared with approximate solutions do not differ in Results of Multiple Scales, Lindstedt Poincare method and we found that the approximate solutions.

References

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