New generalized hyperbolic functions to find exact solution of the nonlinear partial differential equation

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Abstract

In this article, we first time define new functions (called generalized hyperbolic functions) and devise new kinds of transformation (called generalized hyperbolic function transformation) to construct new exact solutions of nonlinear partial differential equations. Based on the generalized hyperbolic function transformation of the generalized KdV equation. We obtain abundant families of new exact solutions of the equation and analyze the properties of this by taking different parameter values of the generalized hyperbolic functions. As a result, we find that these parameter values and the region size of the independent variables affect some solution structure. These solutions may be useful to explain some physical phenomena.

1 Introduction

To construct exact solutions to nonlinear partial differential equations, some important methods have been defined such as Hirota method, tanh-coth method, the exponential function method, (G'/G)-expansion method, the trial equation method, and so on [1-15]. There are a lot of nonlinear evolution equations that are integrated using the various mathematical methods. Soliton solutions, compactons, singular solitons and other solutions have been found by using these approaches. These types of solutions are very important and appear in various areas of applied mathematics. In Section 2, we give the definition and properties of generalized hyperbolic functions. In Section 3, as applications, we obtain exact solution of the generalized KdV equation

$$(u^l)_t + \alpha u(u^n)_x + \beta[u(u^n)_{xx}]_x + \gamma u(u^n)_{xxx} = 0. \tag{1}$$

2 The definition and properties of the symmetrical hyperbolic Fibonacci and Lucas functions

In this section, we will define new functions which named the symmetrical hyperbolic Fibonacci and Lucas functions for constructing new exact solutions of NPDEs, and then study the properties of these functions.

Definition 2.1 Suppose that ξ is an independent variable, p, q and k are all constants. The generalized hyperbolic sine function is

$$\sinh_a(\xi) = \frac{pa^{k\xi} - qa^{-k\xi}}{2}, \tag{2}$$

generalized hyperbolic cosine function is

$$\cosh_a(\xi) = \frac{pa^{k\xi} + qa^{-k\xi}}{2}, \tag{3}$$

generalized hyperbolic tangent function is

$$\tanh_a(\xi) = \frac{pa^{k\xi} - qa^{-k\xi}}{pa^{k\xi} + qa^{-k\xi}}, \tag{4}$$

generalized hyperbolic cotangent function is

$$\coth_a(\xi) = \frac{pa^{k\xi} + qa^{-k\xi}}{pa^{k\xi} - qa^{-k\xi}}, \tag{5}$$

generalized hyperbolic secant function is

$$sech_a(\xi) = \frac{2}{pa^{k\xi} + qa^{-k\xi}}, \tag{6}$$

generalized hyperbolic cosecant function is

$$cosech_a(\xi) = \frac{2}{pa^{k\xi} - qa^{-k\xi}}, \tag{7}$$

the above six kinds of functions are said generalized new hyperbolic functions. Thus we can prove the following theory of generalized hyperbolic functions on the basis of Definition 2.1.

Theorem 2.1. The generalized hyperbolic functions satisfy the following relations:

$$\cosh_a^2(\xi) - \sinh_a^2(\xi) = pq, \tag{8}$$

$$1 - \tanh_a^2(\xi) = pq.sech_a^2(\xi), \tag{9}$$

$$1 - \coth_a^2(\xi) = -pq.cosech_a^2(\xi), \tag{10}$$

$$sech_a(\xi) = \frac{1}{\cosh_a(\xi)}, \tag{11}$$

$$cosech_a(\xi) = \frac{1}{\sinh_a(\xi)}, \tag{12}$$

$$\tanh_a(\xi) = \frac{\sinh_a(\xi)}{\cosh_a(\xi)}, \tag{13}$$

$$\coth_a(\xi) = \frac{\cosh_a(\xi)}{\sinh_a(\xi)}. \tag{14}$$

The following just part of them are proved here for simplification.

Theorem 2.2. The derivative formulae of generalized hyperbolic functions as following

$$\frac{d(\sinh_a(\xi))}{d\xi} = k \ln a \;\cosh_a(\xi), \tag{15}$$

$$\frac{d(\cosh_a(\xi))}{d\xi} = k \ln a \;\sinh_a(\xi), \tag{16}$$

$$\frac{d(\tanh_a(\xi))}{d\xi} = kpq \ln a \; sech_a^2(\xi), \tag{17}$$

$$\frac{d(\coth_a(\xi))}{d\xi} = -kpq \ln a \; cosech_a^2(\xi), \tag{18}$$

$$\frac{d(sech_a(\xi))}{d\xi} = -k \ln a \; sech_a(\xi) \tanh_a(\xi), \tag{19}$$

$$\frac{d(cosech_a(\xi))}{d\xi} = -k \ln a \; cosech_a(\xi) \coth_a(\xi). \tag{20}$$

Proof of (17): According to (15) and (16), we can get

$$\frac{d(\tanh_a(\xi))}{d\xi} = \left(\frac{\sinh_a(\xi)}{\cosh_a(\xi)}\right)' = \frac{(\sinh_a(\xi))' \cosh_a(\xi) - (\cosh_a(\xi))' \sinh_a(\xi)}{\cosh_a^2(\xi)}$$

$$= \frac{k\ln a\ \cosh_a^2(\xi) - k\ln a\ \sinh_a^2(\xi)}{\cosh_a^2(\xi)} = kpqsech_(^2\xi), \tag{21}$$

Similarly, we can prove other differential coefficient formulae in Theorem 2.2.

Remark 2.1. We see that when $p = 1$, $q = 1$, $k = 1$ and $a = e$ in (2)-(7), new generalized hyperbolic function $\sinh_a(\xi)$, $\cosh_a(\xi)$, $\tanh_a(\xi)$, $\coth_a(\xi)$, $sech_a(\xi)$ and $cosech_a(\xi)$, degenerate as hyperbolic function $\sinh(\xi)$, $\cosh(\xi)$, $\tanh(\xi)$, $\coth(\xi)$, $sech(\xi)$ and $cosech(\xi)$, respectively. In addition, when $p = 0$ or $q = 0$ in (2)-(7), $\sinh_a(\xi)$, $\cosh_a(\xi)$, $\tanh_a(\xi)$, $\coth_a(\xi)$, $sech_a(\xi)$ and $cosech_a(\xi)$, degenerate as exponential function $\frac{1}{2}pa^{k(\xi)}$, $\pm\frac{1}{2}qa^{-k(\xi)}$, $2pa^{-k(\xi)}$, $\pm 2qa^{k(\xi)}$ and ± 1, respectively.

References

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