Finite Difference Method for the Integral-Differential Equation of the Hyperbolic Type

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Abstract.

Hyperbolic partial differential equations are used in many branches of physics, engineering and several areas of science, e.g. electromagnetic, electrodynamic, hydrodynamics, elasticity, fluid flow and wave propagation [1, 2, 3]. There is a great deal of work for solving these type of problems numerically and their stability in various functional spaces has received a great deal of importance. However, most of these works are studied in one-dimensional space (see [4, 5, 6, 7]). There are some studies about the numerical solution of two-dimensional hyperbolic equations with the collocation method or rational differential quadrature method [8, 9].

Let Ω be the unit open cube in the n-dimensional Euclidean space $\mathbb{R}^n$ $(0 < x_k < 1,\ 1 \le k \le n)$ with the boundary S, $\overline{\Omega} = \Omega \cup S$. In $[-1,1] \times \Omega$ the mixed problem for the multidimensional integral-differential equation of the hyperbolic type

$$\begin{cases} v_{tt} - \sum\limits_{r=1}^{n} (a_r(x)V_{x_r})_{x_r} = \int\limits_{-t}^{t} \sum\limits_{r=1}^{n} (b_r(p,x)v_{x_r})_{x_r}\, dp + f(t,x), \quad -1 \le t \le 1,\ x = (x_1,\ldots,x_n) \in \Omega, \\ v(t,x) = 0, \quad x \in S,\ -1 \le t \le 1, \\ v(0,x) = \varphi(x),\ v_t(0,x) = \psi(x), \quad x \in \overline{\Omega} \end{cases} \tag{1}$$

is considered. In [10] it was proved that the problem (1) has a unique smooth solution $v(t,x)$ for the smooth functions $a_r(x) \ge \delta > 0$, $r = 1,\ldots,n$, $\varphi(x)$, $\psi(x)$, $x \in \overline{\Omega}$ and $f(t,x)$, $b(t,x)$, $t \in (-1,1)$, $x \in \Omega$. Moreover, the first order of accuracy difference scheme was investigated.

In the present paper the second order of accuracy difference scheme approximately solving the problem (1) is studied. The stability estimates for the solution of this difference scheme are established. Theoretical results are supported by numerical examples.

Keywords: Finite Difference Method; Integral-Differential Equation of the Hyperbolic Type
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